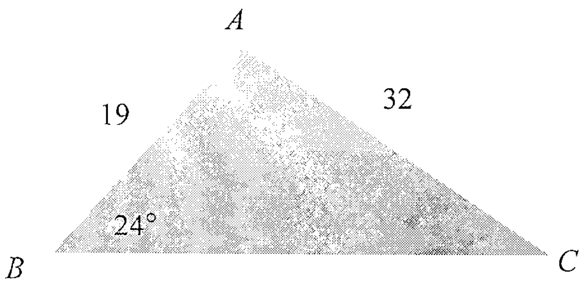
1) (6 points each) Solve the following triangles using the appropriate Law. Be sure to show all necessary work. Round answers to nearest whole number:

a)



$A = 142°$ $a = 48$

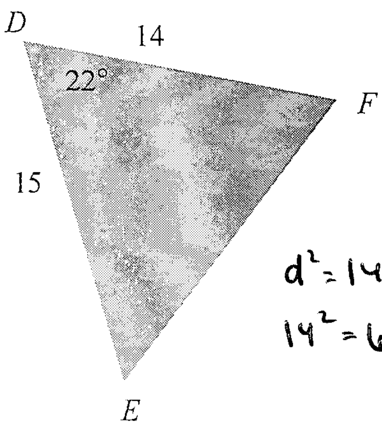
$B = 24°$ $b = 32$

$C = 14°$ $c = 19$

$\frac{\sin C}{19} = \frac{\sin 24°}{32} \Rightarrow C = \sin^{-1}\left(\frac{19 \sin 24°}{32}\right) = 14°$

$A = 180 - (24+14) = 142°$ $\frac{a}{\sin 142°} = \frac{32}{\sin 24°} \Rightarrow a = \frac{32 \sin 142°}{\sin 24°} = 48$

b)



$D = 22°$ $d = 6$

$E = 69°$ $e = 14$

$F = 89°$ $f = 15$

$d^2 = 14^2 + 15^2 - 2(14)(15)\cos 22° \Rightarrow \boxed{d = 6}$

$14^2 = 6^2 + 15^2 - 2(6)(15)\cos E \Rightarrow E = \cos^{-1}\left(\frac{14^2 - 6^2 - 15^2}{-2 \cdot 6 \cdot 15}\right)$

$\approx 69°$

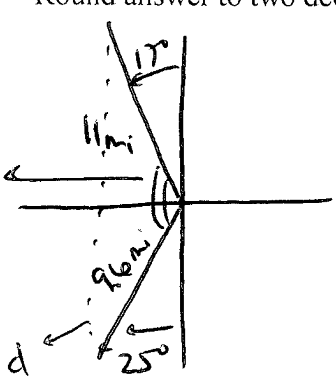
$F = 180 - (22 + 69°) = 89°$

2) (2 points each) Concerning the given information of a triangle, how do you know when to use the Law of Sines versus the Law of Cosines?

Hmm

3) (5 points) Mike announces a test and two students begin to run away from Mike from the same point. One student runs with a bearing of $S25°W$ at 4.8 mph while the other student runs with a bearing of $N17°W$ at 5.5 mph. How far are the students from each other after 2 hours? Round answer to two decimal places. Draw a picture for this scenario.

$\theta = 180 - 17 - 25° = 138°$



$d = \sqrt{9.6^2 + 11^2 - 2(9.6)(11)\cos 138°}$

≈ 19.24mi

19

2

4) (4 points each) Convert… $300°$

a) $7\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$ to standard form:

$= 7\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$= \frac{7}{2} - \frac{7\sqrt{3}}{2}i$

b) $-6\sqrt{3}-6i$ to trigonometric form:

$r = \sqrt{(-6\sqrt{3})^2 + (-6)^2} = \sqrt{144} = 12$

$\cos\theta = \frac{-6\sqrt{3}}{12} = -\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{-6}{12} = -\frac{1}{2}$

$\theta = 210°$ or $\frac{7\pi}{6}$

$12(\cos 210° + i\sin 210°)$

Use #4!

5) (4 points each) For the complex numbers $z_1 = \frac{7}{2} - \frac{7\sqrt{3}}{2}i$ and $z_2 = -6\sqrt{3}-6i$, find the following, using the trigonometric forms and the formula $z_1 \times z_2 = r_1 \times r_2\left[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right]$ for part *a* and $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right]$ for part *b*. Write in standard form.

a) $z_1 \times z_2$

$= 7\cdot 12(\cos(300+210°) + i\sin(300+210°))$

$= 84(\cos 510° + i\sin 510°)$ coterminal

$= 84(\cos 150° + i\sin 150°)$

$= 84\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -42\sqrt{3} + 42i$

b) $\frac{z_1}{z_2}$ $= \frac{7}{12}(\cos(300-210°) + i\sin(300-210°))$

$= \frac{7}{12}(\cos 90° + i\sin 90°)$

$= \frac{7}{12}(0 + 1i) = \frac{7}{12}i$

6) (4 points part *a*; 6 points part *b*) For the complex number $-4+4\sqrt{3}i = 8(\cos 120° + i\sin 120°)$, find the following. For part *a*, use the formula $(a+bi)^n = r^n\left[\cos(n\theta)+i\sin(n\theta)\right]$. For part *b*, use the formula $(a+bi)^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n}+\frac{360°}{n}\cdot k\right)+i\sin\left(\frac{\theta}{n}+\frac{360°}{n}\cdot k\right)\right]$. Write answers in standard form.

a) $\left(-4+4\sqrt{3}i\right)^2$

$= 8^2(\cos(2\cdot 120°) + i\sin(2\cdot 120°))$

$= 64(\cos 240° + i\sin 240°)$

$= 64\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$= -32 - 32\sqrt{3}i$

b) The cube roots of $-4+4\sqrt{3}i$

$= 8^{\frac{1}{3}}\left(\cos\left(\frac{120°}{3} + \frac{360°}{3}\cdot k\right) + i\sin\left(\frac{120°}{3} + \frac{360°}{3}\cdot k\right)\right)$

$= 2(\cos(40° + 120°k) + i\sin(40° + 120°k))$

$k=0$ – $2(\cos 40° + i\sin 40°)$

$k=1$ – $2(\cos 160° + i\sin 160°)$

$k=2$ – $2(\cos 280° + i\sin 280°)$

can't write in standard form easily :)

34

7) (4 points) Convert the rectangular point $(-5,-5)$ to the polar format (r,θ):

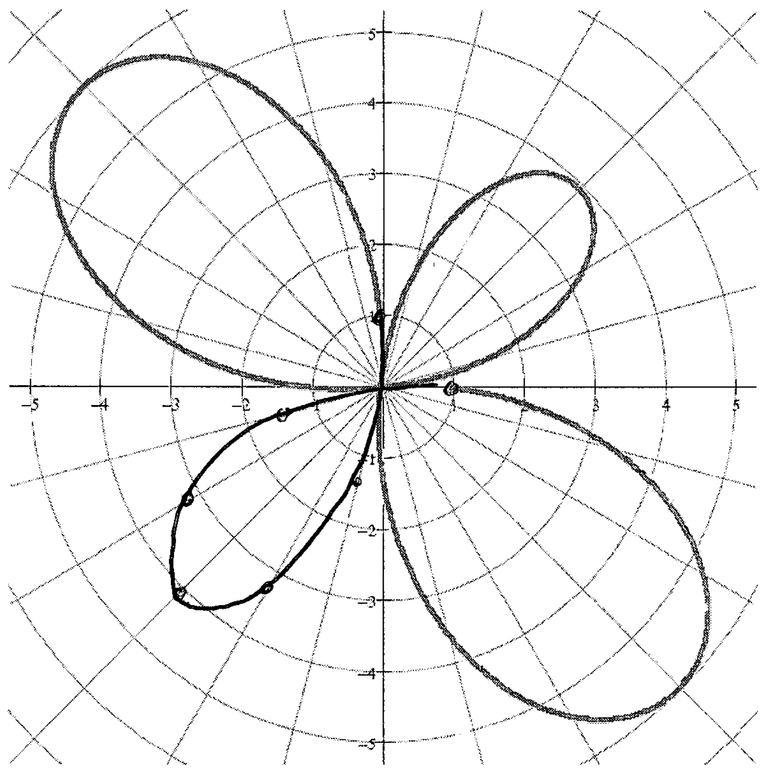
$r = \sqrt{(-5)^2+(-5)^2} = \sqrt{50} = 5\sqrt{2}$

$\left.\begin{array}{l} \cos\theta = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2} \\ \sin\theta = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{array}\right\} \theta = 225^\circ \text{ or } \frac{5\pi}{4}$

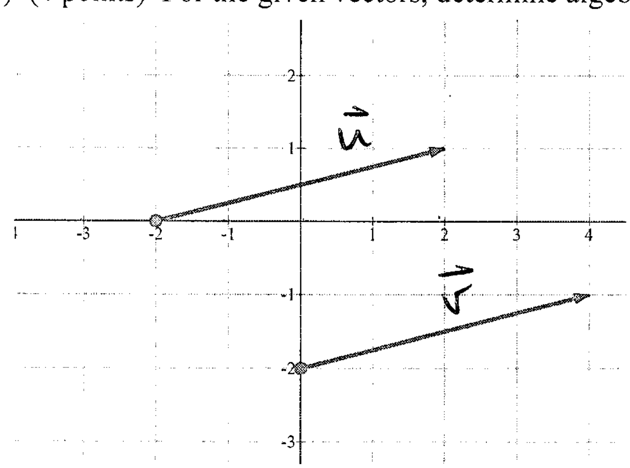
$(5\sqrt{2}, 225^\circ)$ or $\frac{5\pi}{4}$

8) (6 points) Finish the **polar graph** of $r = 1 - 5\sin(2\theta)$ by filling in the chart and ~~plotting points.~~
Round answers to one decimal place:

θ	r
0°	1
15°	-1.5
30°	-3.3
45°	-4
60°	-3.3
75°	-1.5
90°	1



9) (4 points) For the given vectors, determine algebraically if they are equivalent:



$|\vec{u}| = \sqrt{(2-(-2))^2+(1-0)^2} = \sqrt{17}$

$|\vec{v}| = \sqrt{(4-0)^2+(-1-(-2))^2} = \sqrt{17}$

$m_{\vec{u}} = \frac{1-0}{2-(-2)} = \frac{1}{4}$

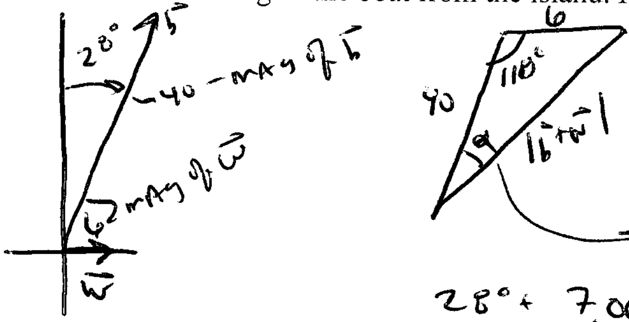
$m_{\vec{v}} = \frac{-1-(-2)}{4-0} = \frac{1}{4}$ ✓

$\vec{u} = \vec{v}$

or you can show they both have component form
$\langle 4, 1 \rangle$

14

10) (10 points) Link is traveling in a boat on a bearing of $N28°E$ from Nintendo Island to his new home of Sony Land. His boat travels at a rate of 20 mph. The water has a current from the west at a rate of 3 mph. After 2 hours, how far is Link from Nintendo Island? Also determine the bearing of the boat from the island. Round to two places.



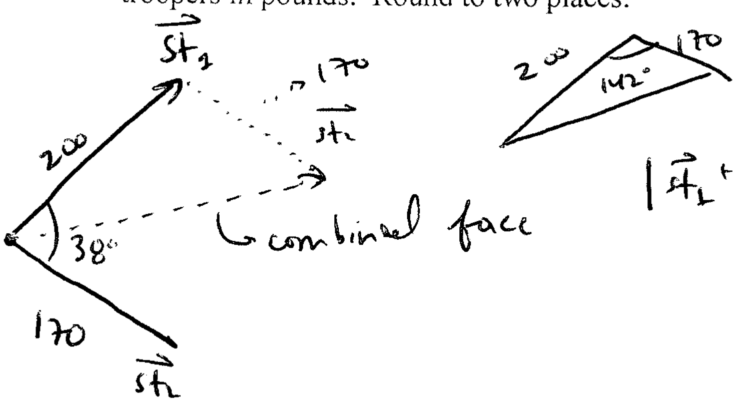
$|\vec{b}+\vec{w}| = \sqrt{6^2+40^2-2(6)(40)\cos 118°}$

$\approx$ 43.14 mi

$\alpha = \cos^{-1}\left(\dfrac{6^2-40^2-43.14^2}{-2(40)(43.14)}\right) \approx 7.06°$

$28° + 7.06 = 35.06° \rightarrow$ N 35.06°E

11) (6 points) In a fit of nerd rage, two storm troopers draw an unruly George Lucas to his movie trailer when started to complain that there wasn't enough Jar Jar in the new *Star Trek* movie. The one storm trooper pulls with a force of 200 pounds and the other pulls with a force of 170 pounds. The angle between the storm troopers is 38°. Determine the combined force of the troopers in pounds. Round to two places.

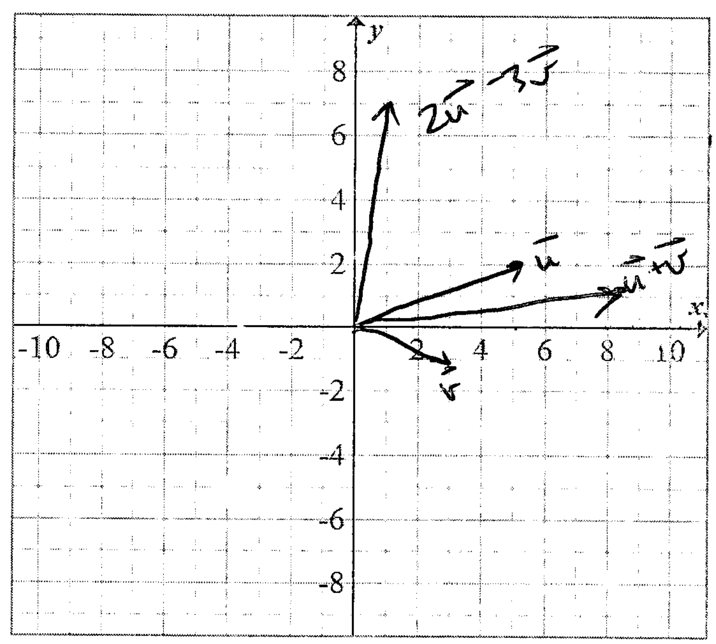


Parallelogram law

$|\vec{st_1}+\vec{st_2}| = \sqrt{200^2+170^2-2(200)(170)\cos 142°}$

$\approx$ 349.98 lbs

12) (5 points) Given the vector $\vec{u}=\langle 5,2\rangle$ and $\vec{v}=\langle 3,-1\rangle$, draw the vectors $\vec{u},\vec{v},\ \vec{u}-3\vec{v}$, and $2\vec{u}-3\vec{v}$:



$\vec{u}+\vec{v} = \langle 8,1\rangle$

$2\vec{u}-3\vec{v} = \langle 1,7\rangle$

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13) (2 points each) Let $\vec{u}=\langle 4,6\rangle$ and $\vec{v}=\langle 4,19\rangle$. Find and simplify:

a) $4\vec{u}-\vec{v}$

$= \langle 16,24\rangle - \langle 4,19\rangle$

$= \boxed{\langle 12, 5\rangle}$

b) $|4\vec{u}-\vec{v}|$

$= \sqrt{12^2+5^2}$

$= 13$

c) The unit vector in the same direction as $4\vec{u}-\vec{v}$:

$\left\langle \frac{12}{13}, \frac{5}{13} \right\rangle$

d) $\vec{u}\bullet\vec{v}$ using $\vec{u}\cdot\vec{v}=u_1\times v_1+u_2\times v_2$:

$= 4\cdot 4 + 6\cdot 19$

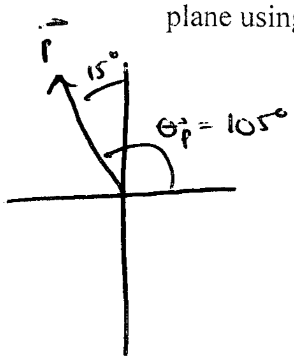
$= 130$

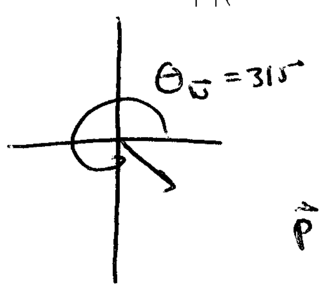
e) The angle between the vectors $\vec{u}$ and $\vec{v}$. Round to two decimal places. Use the formula $\cos\theta=\dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$

$\cos\theta = \dfrac{130}{\sqrt{4^2+6^2}\cdot\sqrt{4^2+19^2}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{130}{\sqrt{52}\cdot\sqrt{377}}\right) \Rightarrow \theta \approx 21.80^\circ$

14) (10 points) An airplane travels on a bearing of $N15°W$ with an airspeed of 600 mph. A wind is blowing **from the northwest** direction at a speed of 40 mph. Find the ground speed of the plane using the formula $\vec{v}=|\vec{v}|(\cos\theta\vec{i}+\sin\theta\vec{j})$. Round to two decimal places:





$\vec{p} = 600(\cos 105°\,\vec{i} + \sin 105°\,\vec{j})$

$\vec{w} = 40(\cos 315°\,\vec{i} + \sin 315°\,\vec{j})$

$\vec{p}+\vec{w} = \underbrace{(600\cos 105° + 40\cos 315°)}_{a}\,\vec{i} + \underbrace{(600\sin 105° + 40\sin 315°)}_{b}\,\vec{j}$

$|\vec{p}+\vec{w}| = \sqrt{a^2+b^2} \approx 565.71$ mph

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