1) (4 points each) Simplify:

a) $\tan x(\tan x + \cot x)$

$= \tan^2 x + \tan x \cdot \cot x$

$= \tan^2 x + 1$

$= \boxed{\sec^2 x}$

b) $\dfrac{\tan x}{\tan x + \cot x}$

$= \dfrac{\frac{\sin x}{\cos x}}{\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)} \cdot \dfrac{\sin x \cos x}{\sin x \cos x}$

$= \dfrac{\sin^2 x}{\sin^2 x + \cos^2 x} = \dfrac{\sin^2 x}{1} = \boxed{\sin^2 x}$

2) (4 points each) Find the exact value of $\cos\frac{\pi}{12}$ using the given methods.

a) A Sum or Difference Formula:

$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\dfrac{\sqrt{2}+\sqrt{6}}{4}}$

b) A Half Angle Formula:

$\cos \pi/12 = \cos \frac{\frac{\pi}{6}}{2}$

$= \sqrt{\dfrac{1 + \cos \pi/6}{2}} = \sqrt{\dfrac{1 + \sqrt{3}/2}{2}}$

$= \boxed{\dfrac{\sqrt{2+\sqrt{3}}}{2}}$

oops!

c) Using your answer from either part *a* or *b* above, explain how you can find the exact value of $\cos$ ~~tan~~ $\left(\frac{13\pi}{12}\right)$ by using $\frac{\pi}{12}$ as a reference angle:

Hmmm. . .

3) (5 points each) Simplify:

a) $\dfrac{\sin(\alpha+\beta) - \sin(\alpha-\beta)}{\cos(\alpha+\beta) - \cos(\alpha-\beta)}$

$= \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta - (\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{\cos\alpha\cos\beta - \sin\alpha\sin\beta - (\cos\alpha\cos\beta + \sin\alpha\sin\beta)}$

$= \dfrac{2\cos\alpha\cancel{\sin\beta}}{-2\sin\alpha\cancel{\sin\beta}} = \boxed{-\cot\alpha}$

b) $\dfrac{\sin^2\alpha}{\tan^2\alpha} - \dfrac{\tan^2\alpha}{\sec^2\alpha} = \dfrac{\sin^2\alpha}{\frac{\sin^2\alpha}{\cos^2\alpha}} - \dfrac{\frac{\sin^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}}$

$= \cos^2\alpha - \sin^2\alpha$

$= \boxed{\cos 2\alpha}$

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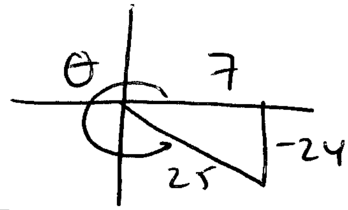

4) (5 points each) Given $\cos\theta = \frac{7}{25}$, where θ is in Quadrant IV, find the exact values for…

a) $\sin(2\theta)$

$= 2\sin\theta\cos\theta$

$= 2\left(-\frac{24}{25}\right)\left(\frac{7}{25}\right) = \boxed{-\frac{336}{625}}$

b) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \left(\frac{7}{25}\right)^2 - \left(-\frac{24}{25}\right)^2$

$= \boxed{-\frac{527}{625}}$

c) $\tan(2\theta)$

$= \frac{\sin 2\theta}{\cos 2\theta} = \frac{-336/625}{-527/625}$

$= \boxed{\frac{336}{527}}$

d) The Quadrant where 2θ resides. Explain why:

Q3 $\sin 2\theta < 0$

$\cos 2\theta < 0$

$\tan 2\theta > 0$

5) (6 points part *a*, 9 points part *b*) Prove the following identities:

a) $\tan^2\theta\sin^2\theta = \tan^2\theta + \cos^2\theta - 1$

$\tan^2\theta(1-\cos^2\theta)$

$\tan^2\theta - \tan^2\theta\cos^2\theta$

$\tan^2\theta - \sin^2\theta$

$\tan^2\theta - (1-\cos^2\theta)$

$\tan^2\theta - 1 + \cos^2\theta$ ■

b) $4\cos^2 x - 4 + \sec^2 x = \cos^2 x - 2\sin^2 x + \sin^2 x\tan^2 x$

$4\cos^2 x - 4 + \frac{1}{\cos^2 x}$

$\frac{4\cos^4 x - 4\cos^2 x + 1}{\cos^2 x}$

$\frac{(2\cos^2 x - 1)^2}{\cos^2 x}$

double angle for cosine

$\frac{(\cos^2 x - \sin^2 x)^2}{\cos^2 x}$

$\frac{\cos^4 x - 2\cos^2 x\sin^2 x + \sin^4 x}{\cos^2 x}$

$= \cos^2 x - 2\sin^2 x + \sin^2 x\tan^2 x$ ■

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6) (4 points each) Find the exact values or explain why it does not exist:

a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{5\pi}{6}$

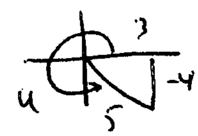
b) $\cos\left(\cos^{-1}(-1.1)\right)$

DNE. -1.1 isn't in the domain of cosine inverse

c) $\cos^{-1}\left(\cos\left(-\frac{2\pi}{3}\right)\right)$

$= \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

d) $\sin\left(\underbrace{\sin^{-1}\left(-\frac{4}{5}\right)}_{u} - \underbrace{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)}_{v}\right)$



$= \sin(\sin^{-1}(-\frac{4}{5}))\cos(\cos^{-1}(-\frac{\sqrt{3}}{2})) - \cos(\sin^{-1}(-\frac{4}{5}))\sin($ $\cos^{-1} -\frac{\sqrt{3}}{2}$

$= (-\frac{4}{5})(-\frac{\sqrt{3}}{2}) - (\frac{3}{5})(\frac{1}{2}) = \frac{4\sqrt{3}-\sqrt{3}}{10}$

7) (6 points each) Solve for the variable:

a) $\cos^2 x - 1 = 0$

$\cos^2 x = 1 \Rightarrow \cos x = \pm 1$

$x = 0 + k\cdot\pi,\ k \in \mathbb{Z}$

b) $\sin(2x) = \frac{\sqrt{3}}{2}$ on $[0, 2\pi)$

$2x = \frac{\pi}{3} + 2\pi k \Rightarrow x = \frac{\pi}{6} + \pi k$

$2x = \frac{2\pi}{3} + 2\pi k \Rightarrow x = \frac{\pi}{3} + \pi k$

$x \in \left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}\right\}$

8) Fill in the blank using interval notation:

	sin x *	cos x *	tan x *	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain	this answer left blank for fun					
Range						

*Write the domain restrictions for these three functions.

9) (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

perchè no?

Extra Credit:

Simplify: $-\sin\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}+\alpha\right)+\cos\left(\frac{\pi}{3}-\alpha\right)\cos\left(\frac{\pi}{3}+\alpha\right)$

$-\frac{1}{2}$

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