

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

- ❖ Write your name below on the space provided.
- ❖ This test has a total of 6 pages.
- ❖ Work the problem in the space provided. If you need more space, write on the back of the test.
- ❖ To insure maximum credit, show your work. In general, full credit will not be given for unsupported answers.
- ❖ Look only at your test. Don't give me the impression that you are cheating.
- ❖ Be sure to write neatly. If I cannot read what was written, do not expect the problem to be graded. A pencil must be used on all tests. Otherwise, the test will not be graded.
- ❖ If you finish early, go over the test again.

Good luck!

Number	Maximum	Score
1	8	
2	12	
3	10	
4	20	
5	15	
6	16	
7	12	
8	5	
9	2	
Total	100	

Name _____

CIRCLE FINAL ANSWERS

Feel free to tear off the last two pages.
I do not need them returned!

1) (4 points each) Simplify:

a) $\tan x(\tan x + \cot x)$

b) $\frac{\tan x}{\tan x + \cot x}$

2) (4 points each) Find the exact value of $\cos \frac{\pi}{12}$ using the given methods.

a) A Sum or Difference Formula:

b) A Half Angle Formula:

c) Using your answer from either part *a* or *b* above, explain how you can find the exact value of $\tan\left(\frac{13\pi}{12}\right)$ by using $\frac{\pi}{12}$ as a reference angle:

3) (5 points each) Simplify:

a) $\frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{\cos(\alpha + \beta) - \cos(\alpha - \beta)}$

b) $\frac{\sin^2 \alpha}{\tan^2 \alpha} - \frac{\tan^2 \alpha}{\sec^2 \alpha}$

4) (5 points each) Given $\cos\theta = \frac{7}{25}$, where θ is in Quadrant IV, find the exact values for...

a) $\sin(2\theta)$

b) $\cos(2\theta)$

c) $\tan(2\theta)$

d) The Quadrant where 2θ resides. Explain why:

5) (6 points part *a*, 9 points part *b*) Prove the following identities:

a) $\tan^2\theta \sin^2\theta = \tan^2\theta + \cos^2\theta - 1$

b) $4\cos^2x - 4 + \sec^2x = \cos^2x - 2\sin^2x + \sin^2x \tan^2x$

6) (4 points each) Find the exact values or explain why it does not exist:

a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

b) $\cos(\cos^{-1}(-1.1))$

c) $\cos^{-1}\left(\cos\left(-\frac{2\pi}{3}\right)\right)$

d) $\sin\left(\sin^{-1}\left(-\frac{4}{5}\right) - \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

7) (6 points each) Solve for the variable:

a) $\cos^2 x - 1 = 0$ (Hint: You need a k)

b) $\sin(2x) = \frac{\sqrt{3}}{2}$ on $[0, 2\pi)$

8) Fill in the blank using interval notation:

	$\sin x$ *	$\cos x$ *	$\tan x$ *	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain						
Range						

*Write the domain restrictions for these three functions.

9) (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

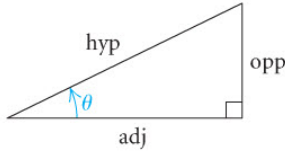
Extra Credit:

Simplify: $-\sin\left(\frac{\pi}{3} - \alpha\right)\sin\left(\frac{\pi}{3} + \alpha\right) + \cos\left(\frac{\pi}{3} - \alpha\right)\cos\left(\frac{\pi}{3} + \alpha\right)$

Trigonometry

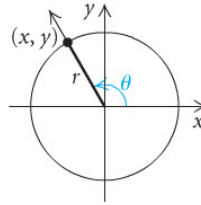
Trigonometric Functions

Acute Angles



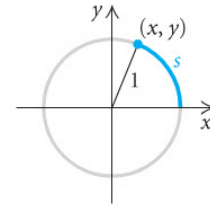
$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}}, & \csc \theta &= \frac{\text{hyp}}{\text{opp}}, \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}}, & \sec \theta &= \frac{\text{hyp}}{\text{adj}}, \\ \tan \theta &= \frac{\text{opp}}{\text{adj}}, & \cot \theta &= \frac{\text{adj}}{\text{opp}}\end{aligned}$$

Any Angle



$$\begin{aligned}\sin \theta &= \frac{y}{r}, & \csc \theta &= \frac{r}{y}, \\ \cos \theta &= \frac{x}{r}, & \sec \theta &= \frac{r}{x}, \\ \tan \theta &= \frac{y}{x}, & \cot \theta &= \frac{x}{y}\end{aligned}$$

Real Numbers



$$\begin{aligned}\sin s &= y, & \csc s &= \frac{1}{y}, \\ \cos s &= x, & \sec s &= \frac{1}{x}, \\ \tan s &= \frac{y}{x}, & \cot s &= \frac{x}{y}\end{aligned}$$

Basic Trigonometric Identities

$$\begin{aligned}\sin(-x) &= -\sin x, \\ \cos(-x) &= \cos x, \\ \tan(-x) &= -\tan x,\end{aligned}$$

$$\begin{aligned}\tan x &= \frac{\sin x}{\cos x}, \\ \cot x &= \frac{\cos x}{\sin x},\end{aligned}$$

$$\begin{aligned}\csc x &= \frac{1}{\sin x}, \\ \sec x &= \frac{1}{\cos x}, \\ \cot x &= \frac{1}{\tan x}\end{aligned}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1, \\ 1 + \cot^2 x &= \csc^2 x, \\ 1 + \tan^2 x &= \sec^2 x\end{aligned}$$

Double-Angle Identities

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x, \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \\ &= 2 \cos^2 x - 1, \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$

Identities Involving $\pi/2$

$$\begin{aligned}\sin(\pi/2 - x) &= \cos x, \\ \cos(\pi/2 - x) &= \sin x, & \sin(x \pm \pi/2) &= \pm \cos x, \\ \tan(\pi/2 - x) &= \cot x, & \cos(x \pm \pi/2) &= \mp \sin x\end{aligned}$$

Half-Angle Identities

$$\begin{aligned}\sin \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{2}}, & \cos \frac{x}{2} &= \pm \sqrt{\frac{1 + \cos x}{2}}, \\ \tan \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}\end{aligned}$$

Sum and Difference Identities

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v, \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v, \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

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Positive:
Negative:

Positive:
Negative:

