$4p = -12 \Rightarrow p = -3$

1) (5 points) Fill in the information for the parabola $(y-2)^2 = -12(x+1)$:

i) $h =$ __-1__

ii) $k =$ __2__

iii) $p =$ __-3__

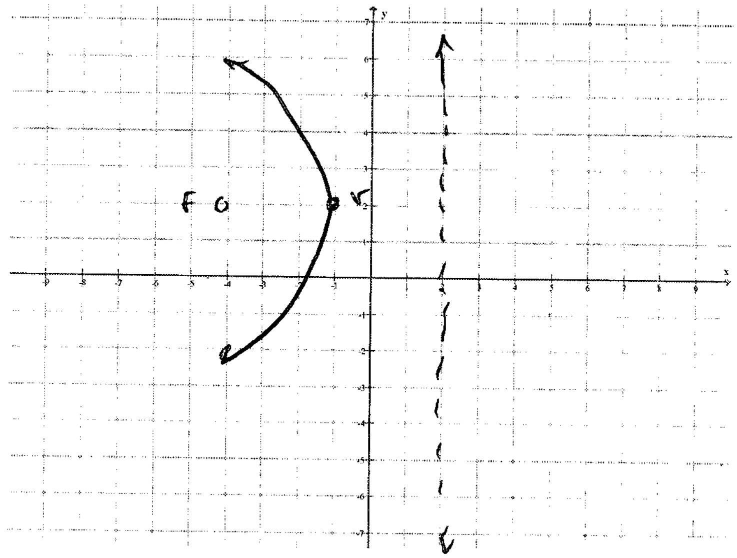
Write iv and v as ordered pairs:

iv) Center __$(-1,2)$__

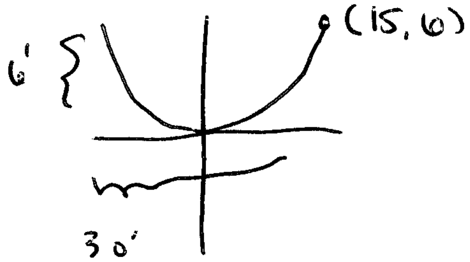
v) Focus __$(-4,2)$__

vi) Directrix __$x=2$__

vi) Sketch the graph:



2) (3 points) Determine the location of the receiver for a 30 foot tall, 6 foot deep parabolic satellite dish assuming the receiver is to be at installed at the foci of the parabola. Give your answer in feet from the vertex of the dish:

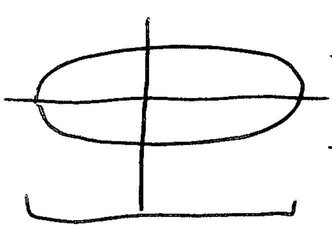


$x^2 = 4py$

$15^2 = 4p(6) \Rightarrow p = \frac{225}{24} = 9.375\text{ ft}$

3) (3 points each) Statuary Hall, also known as the Whispering Gallery, is an elliptical room in the United States Capitol in Washington D.C. where a person standing at one focus of the room can hear even a whisper spoken by a person standing at the other focus. Statuary Hall is 46 feet wide and 97 feet long. Assuming a horizontal ellipse, find…

a) The equation of the ellipse for the room:



$46' \Rightarrow b = 23'$

$97' \Rightarrow a = 48.5'$

$$\frac{x^2}{48.5^2} + \frac{y^2}{23^2} = 1$$

2352.25   529

b) The location of the foci from the center of the room. Round to two decimal places:

$c^2 = a^2 - b^2$

$c^2 = 2352.25 - 529$

$\Rightarrow c \approx 42.70\text{ ft}$

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subtract

4) (3 points each) For the ellipse $25x^2 - 100x + 16y^2 + 96y + 244 = 400$

a) Rewrite the equation by completing the square for both variables:

$$25(x^2 - 4x + 4) + 16(y^2 + 6y + 9) = 156 + 100 + 144$$

$$25(x-2)^2 + 16(y+3)^2 = 400$$

$$\boxed{\frac{(x-2)^2}{16} + \frac{(y+3)^2}{25} = 1}$$

b) Find the **exact values** for the following:

i) $h =$ 2

ii) $k =$ −3

iii) $a =$ 5

iv) $b =$ 4

v) $c =$ 3

Write as ordered pairs:

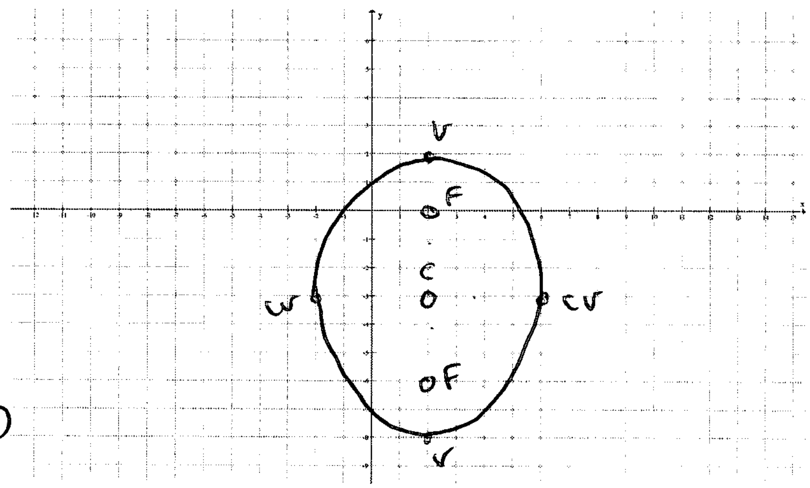
vi) Center (2, −3)

vii) Vertices (2, 2) (2, −8)

viii) Co-Vertices (6, −3) (−2, −3)

ix) Foci (2, 0) (2, −6)

c) Sketch the graph:



5) (6 points) Fill in the information for the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$:

i) $h =$ 0

ii) $k =$ 0

iii) $a =$ 3

iv) $b =$ 4

v) $c =$ 5

Write as ordered pairs:

vi) Center (0, 0)

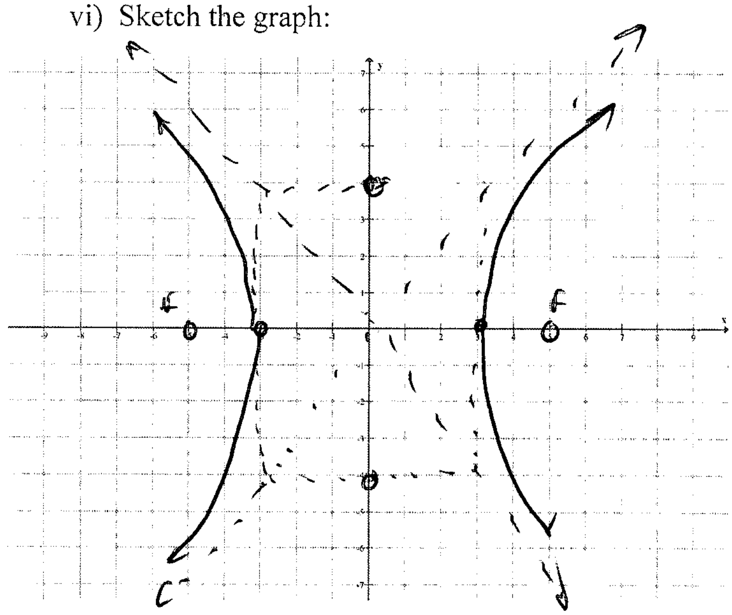
vii) Vertices (3, 0), (−3, 0)

viii) Co-Vertices (0, 4), (0, −4)

ix) Foci (5, 0), (−5, 0)

x) Asymptotes $y = \frac{4}{3}x$, $y = -\frac{4}{3}x$

vi) Sketch the graph:



15

6) (5 points each) Solve the following systems. For part *b*, shade your final in the darkest:

a) $\begin{cases} xy = 3 \\ x + 3y = -10 \end{cases}$ $\Rightarrow$ $x = \frac{3}{y}$

$\frac{3}{y} + 3y = -10$

$3 + 3y^2 = -10y^2$

$3y^2 + 10y + 3 = 0$

$(3y+1)(y+3) = 0$

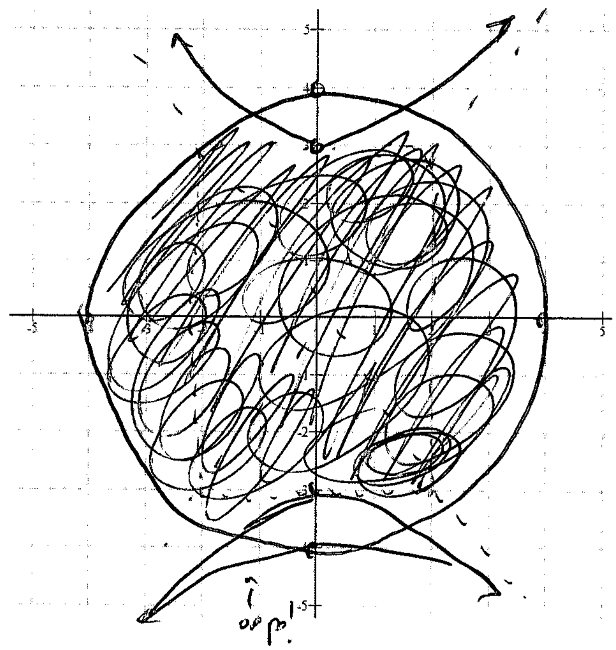
$y = -\frac{1}{3}$ $\quad$ $y = -3$

$x = 3/(-1/3)$ $\quad$ $x = 3/-3$

$x = -9$ $\quad$ $x = -1$

$(-9, -\frac{1}{3})$
$(-1, -3)$

b) $\begin{cases} x^2 + y^2 \le 16 \\ \frac{y^2}{9} - \frac{x^2}{4} \le 1 \end{cases}$ circle / hyperbola



7) (3 points) What is a sequence?

a dweam within a dweam

8) (4 points each) Find the first four terms of the following sequences using fractions as necessary. Determine if they are arithmetic, geometric, or neither. If it is arithmetic, determine the common difference. If it is geometric, determine the common ratio.

a) $\{5n-2\}$

$n=1$ $\quad 5(1)-2 = 3$
$n=2$ $\quad 5(2)-2 = 8$
$n=3$ $\quad 5(3)-2 = 13$
$n=4$ $\quad 5(4)-2 = 18$
$n=5$ $\quad 5(5)-2 = 23$

Arithmetic
$d = 5$

b) $a_1 = 27$, $a_{n+1} = \frac{2}{3}a_n$, $n \ge 1$

$a_1 = 27$
$a_2 = \frac{2}{3} \cdot 27 = 18$
$a_3 = \frac{2}{3} \cdot 18 = 12$
$a_4 = \frac{2}{3} \cdot 12 = 8$
$a_5 = 2/3 \cdot 8 = 16/3$

geometric
$r = \frac{2}{3}$

9) (4 points each) Given the sequence below, find the general term a_n. For part *a* only, use the formula $a_n = a_1 + (n-1)d$ and simplify:

a) 8, 5, 2, −1,...

$a_n = 8 + (n-1)(-3)$

$a_n = -3n + 11$

b) $\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{3}}{2}, \frac{2}{\sqrt{5}}, \frac{\sqrt{5}}{\sqrt{6}}, \ldots$

$a_n = \frac{\sqrt{n}}{\sqrt{n+1}}$

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10) (3 points) Find the sum $\sum_{k=4}^{7}(k+1)(k-1)$. Be sure to write out the individual terms:

$= (4+1)(4-1) + (5+1)(5-1) + (6+1)(6-1) + (7+1)(7-1)$

$= 122$

11) (3 points each) Write the follow series in sigma notation. Refer to problem 9:

a) $8+5+2+(-1)+...+(-28)$ Prob 9a

arithmetic

$-3n+11=-28$

$\Rightarrow n=13$

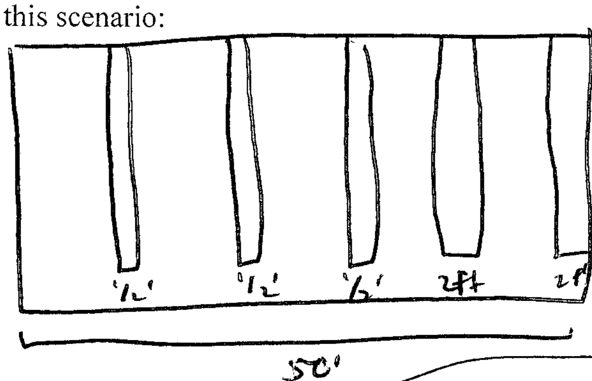
$$\sum_{k=1}^{13}(-3k+11)$$

b) $\frac{1}{\sqrt{2}}+\frac{\sqrt{2}}{\sqrt{3}}+\frac{\sqrt{3}}{2}+\frac{2}{\sqrt{5}}+\frac{\sqrt{5}}{\sqrt{6}}+...+\frac{7}{\sqrt{50}}$

$$\sum_{k=1}^{49}\frac{\sqrt{k}}{\sqrt{k+1}}$$

12) (4 points each) A 50-foot long parking lot is to be painted so that it contains 5 equal-width parking spots. The right-most spot is designated for handicap only and will be flanked by a 2-foot wide line on each side to allow for the doors to open fully. The other 4 spots will be separated by a ½ foot wide line. There is no line drawn at the left edge of the parking lot.

a) Draw a picture for this scenario:



$\frac{50-(2\cdot 2+3\cdot \frac{1}{2})}{5}$

$= 8.7$ ft wide spaces

b) How wide is each of the 5 parking spots?

oops!
too much coffee

8.7ft

c) Starting your count from the left-most edge of the parking lot, determine the location of the left-edge of the ½ foot wide lines:

8.7', 17.9', 27.1'

$+\frac{1}{2}+8.7'$ $+\frac{1}{2}+8.7'$

21

$n = 78 - 42 + 1 = 37$

13) (4 points each) Evaluate the sums using either formula: $S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$ or $S_n = \frac{n}{2}\left(a_1 + a_n\right)$.

Write answers as improper fractions as needed:

a) $\sum_{k=1}^{500}(10k+2)$

$a_1 = 10(1)+2 = 12$

$a_{500} = 10(500)+2 = 5002$

$S_{500} = \frac{500}{2}(12+5002) = 1{,}253{,}500$

b) $\sum_{k=42}^{78}\left(\frac{k-3}{6}\right)$

$a_1 = \frac{42-3}{6} = \frac{13}{2}$

$a_{37} = \frac{78-3}{6} = \frac{25}{2}$

$= \frac{37}{2}\left(\frac{13}{2} + \frac{25}{2}\right) = \frac{703}{2}$

$= 351.5$

14) (4 points) Short answer: Why can't you find an infinite sum of terms of an arithmetic sequence, but you can under certain constrictions of a geometrics sequence.

cuz

15) (4 points) Find the sum $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots + \frac{128}{2187}$ using $S_n = a_1 \frac{1-r^n}{1-r}$. You may need to refer to number 9. Write answer as an improper fraction using MATH > FRAC on your calculator:

$\frac{128}{2187} = 1\left(\frac{2}{3}\right)^{n-1}$

$\left(\frac{2}{3}\right)^7 = \left(\frac{2}{3}\right)^{n-1}$

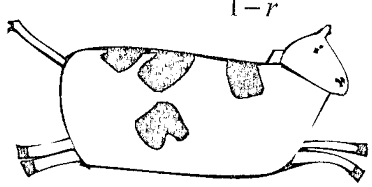
$n-1 = 7$

$n = 8$

$S_8 = 1\left(\frac{1-(2/3)^8}{1-\frac{2}{3}}\right)$

$= \frac{6305}{2187}$

16) (4 points) A daredevil rubber cow jumps from a 450 foot building and always rebounds $\frac{1}{5}$ of the distance fallen. How far does the cow travel vertically? In your answer be sure to use the formula $S_\infty = \frac{a_1}{1-r}$.



Rubber cows produce milkshakes.
Milkshakes, in turn, bring boys to the yard.
Therefore, rubber cows bring boys to the yard.

$450 + 90 + 90 + 18 + 18 + \ldots$

$= 450 + 2\left(\frac{90}{1-1/5}\right)$

$= 675$ ft

20