1) (3 points each) Find the inverse of the following functions:

a) $\{(8,4),(0,7),(12,9),(6,6)\}$

$\{(4,8),(7,0),(9,12),(6,6)\}$

b) $f(x)=\frac{2}{x-3}$

① $y=\frac{2}{x-3}$

② $x=\frac{2}{y-3}$

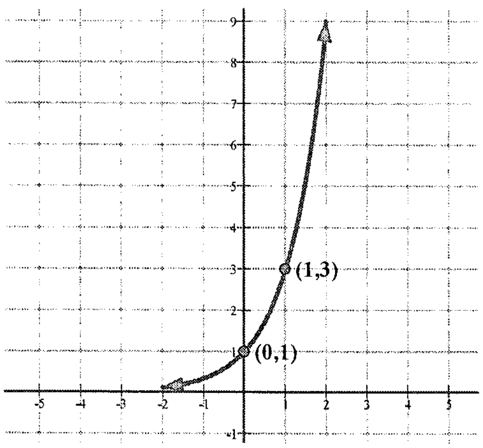
③ $xy-3x=2$

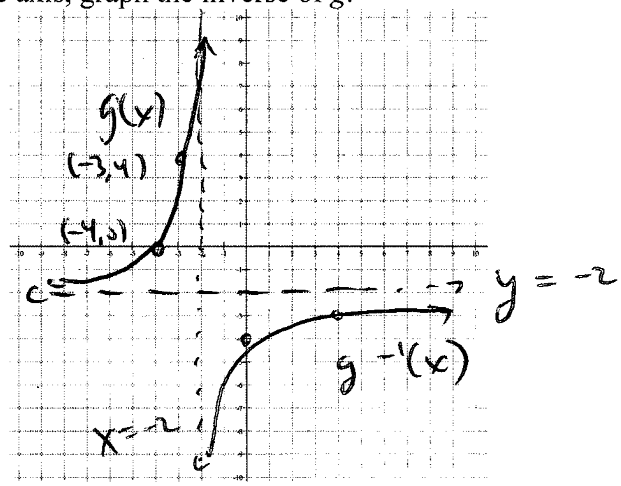
$xy=2+3x$

$y=\frac{2+3x}{x}$

④ $\boxed{f^{-1}(x)=\frac{2+3x}{x}}$

2) (8 points) Graph $g(x)=2\cdot 3^{x+4}-2$ by transforming the given function $y=3^x$. Be sure to move and label the given points and asymptotes. On the same axis, graph the inverse of g.





3) Extra credit (2 points) What was the inverse function graphed in #2? As a hint, remember what the inverse of an exponential function is.

$$g^{-1}(x)=\log_3\left(\frac{x+2}{2}\right)-4$$

4) (3 points) Write as one logarithm $9\log_4 x+5\log_4 y-10\log_4 z$:

$$=\log_4 x^9+\log_4 y^5-\log_4 z^{10}$$

$$=\log_4\left(\frac{x^9y^5}{z^{10}}\right)$$

5) (5 points) Given that $\ln x=5$, $\ln y=10$, and $\ln z=-5$, find the exact value for $\ln\sqrt[5]{\frac{x^4y}{z^5}}$:

$$\ln\sqrt[5]{\frac{x^4y}{z^5}}=\frac{1}{5}\left(\ln\frac{x^4y}{z^5}\right)=\frac{1}{5}\left(\ln x^4+\ln y-\ln z^5\right)$$

$$=\frac{1}{5}\left(4\ln x+\ln y-5\ln z\right)=\frac{1}{5}(4(5)+10-5(-5))=11$$

22/24

6) (2 points each) Finish the explanation of the given equations. **Do not solve the equations**.

a) $5^{4x-7} = 125$ is a one-to-one exponential equation because…

variable is in the exponent and both sides can be written as the same base

b) $\log_{17}(2x+8) = 16$ is not a one-to-one logarithmic equation because…

variable is in a log's argument and cannot (easily) be written as log = log.

7) (5 points each) Solve for the variable. Be sure to find the exact value.

a) $8^{2x-1} = 16^{x+4}$

$(2^3)^{2x-1} = (2^4)^{x+4}$

$2^{6x-3} = 2^{4x+16}$

$6x-3 = 4x+16 \Rightarrow x = \frac{19}{2}$

b) $10e^{4x+1} = 13$

$e^{4x+1} = \frac{13}{10}$

$\ln\frac{13}{10} = 4x+1$

$\frac{\ln\frac{13}{10} - 1}{4} = x$

c) $\log_2(x+1) - \log_2(x+2) = \log_2 8$

$\log_2 \frac{x+1}{x+2} = \log_2 8$

$\frac{x+1}{x+2} = 8$

$x+1 = 8x+16 \Rightarrow x = -\frac{15}{7}$

$\varnothing$ not in domain

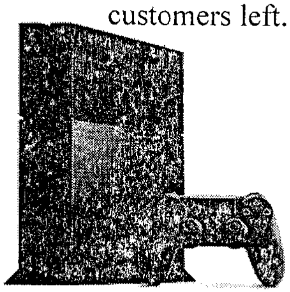
d) $\ln(3x+4) + 3 = 12$

$\ln(3x+4) = 9$

$e^9 = 3x+4$

$\frac{e^9 - 4}{3} = x$

8) (6 points) At a conference, when it was revealed that the Xbox One would not allow you to share games with friends, the number of Xbox owners began to decay exponentially. Using $P(t) = P_0 e^{-kt}$ where P is in **millions** of Xbox customers and t is the number of hours since the beginning of the conference, determine the exact value for the decay rate k if there were 2.37 million customers in the beginning of the event and 5 hours later, there are only 1 million customers left.



University studies have shown that the PS4 is totes cooler than the Xbox One.

$1 = 2.37 e^{-k \cdot 5}$

$\frac{1}{2.37} = e^{-5k}$

$\ln\frac{1}{2.37} = -5k$

$k = \frac{\ln\frac{1}{2.37}}{-5}$

/30

9) (2 points part *a*, 4 points part *b*) Consider the following problem.

At a college production of Much Ado about Math, *400 tickets were sold. The ticket prices were \$8, \$10, and \$12, and the total income from the ticket sales was \$3700. How many tickets of each type were sold if the combined number of \$8 and \$10 tickets sold was 7 times the number of \$12 tickets sold?*

a) Name and define variables:

x = # of \$8 tix sold
y = # of \$10 tix sold
z = # of \$12 tix sold

b) Set up the system but **do not solve it.**

$$\begin{cases} x+y+z=400 \\ 8x+10y+12z=3700 \\ x+y=7z \end{cases}$$

10) (9 points each) Solve the following system using the method listed:

$$\begin{cases} x-4y+2z=-4 \\ 4x-15y+8z=2 \\ -2x+9y-10z=8 \end{cases}$$

a) Elimination:

① $-4x+16y-8z=16$
$4x-15y+8z=2$
$y=18$

② $2x-8y+4z=-8$
$-2x+9y-10z=8$
$y-6z=0$

③ $y=18$

④ $y-6z=0$
$18=6z$
$z=3$

$x-4y+2z=-4$
$x-4(18)+2(3)=-4$
$x=62$

$(62,18,3)$

b) Gauss-Jordan Method:

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & -4 \\ 4 & -15 & 8 & 2 \\ -2 & 9 & -10 & 8 \end{array}\right] \xrightarrow[2R_1+R_3\to R_3]{-4R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -4 & 2 & -4 \\ 0 & 1 & 0 & 18 \\ 0 & 1 & -6 & 0 \end{array}\right] \xrightarrow[-R_2+R_3\to R_3]{4R_2+R_1\to R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 68 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & -6 & -18 \end{array}\right] \xrightarrow[-2R_3+R_1\to R_1]{-\frac{1}{6}R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 62 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 1 & 3 \end{array}\right] (62,18,3)$$

11) (2 points) Verify that you made absolutely sure that your answer to 10a is the same as in 10b by signing your name here Taco wednesday. You will not receive the credit if the work does not support the same answer.

30

12) (2 points) Short answer: Why are logarithms necessary?

Cuz

13) (10 points each) Decompose into partial fractions:

a) $\frac{x+10}{x^2-4x-12} = \frac{A}{x-6} + \frac{B}{x+2}$

$x+10 = A(x+2) + B(x-6)$

$x+10 = Ax+2A$

$\begin{cases} A+B=1 \\ 2A-6B=10 \end{cases} \Rightarrow \begin{array}{l} A=2 \\ B=-1 \end{array}$

$\frac{2}{x-6} + \frac{-1}{x+2}$

b) $\frac{x^2+2x+7}{(x^2+2)(x+1)} = \frac{Ax+B}{x^2+2} + \frac{C}{x+1}$

$x^2+2x+7 = (Ax+B)(x+1) + C(x^2+2)$

$x^2+2x+7 = Ax^2+Bx+Ax+B+Cx^2+2C$

$\begin{cases} A + C = 1 \\ A+B = 2 \\ B+2C = 7 \end{cases} \Rightarrow \begin{array}{l} A=-1 \\ B=3 \\ C=2 \end{array}$

$\frac{-x+3}{x^2+2} + \frac{2}{x+1}$

22