

1) (2 points each) For the given points  $(6, -2)$  and  $(1, 10)$ , find...

a) The distance between them:

$$d = \sqrt{(1-6)^2 + (10-(-2))^2}$$
$$= \sqrt{169} = 13$$

b) Their midpoint:

$$\left(\frac{6+1}{2}, \frac{-2+10}{2}\right) = \left(\frac{7}{2}, 4\right)$$

c) Find the equation of the circle where  $(6, -2)$  and  $(1, 10)$  are endpoints of a diameter of the circle:

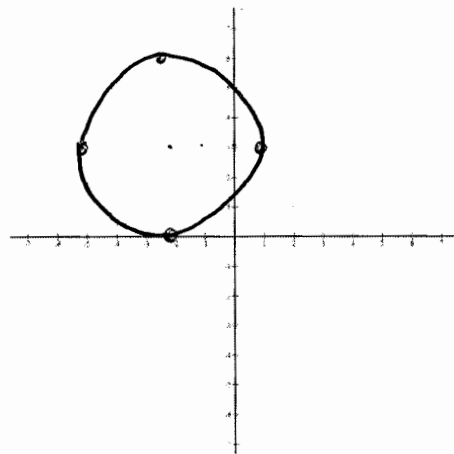
$$\left(x - \frac{7}{2}\right)^2 + (y - 4)^2 = \left(\frac{13}{2}\right)^2 = \frac{169}{4}$$

2) (2 points each) For the circle  $(x+2)^2 + (y-3)^2 = 9$ ...

a) Find the center and radius:

Center:  $(-2, 3)$   
radius: 3

b) Sketch a graph:



3) (2 points) Find the equation of a line in slope-intercept form that passes through the point  $(1, -5)$  and is perpendicular to  $4x - 9y = 12$ :

$$4x - 9y = 12 \Rightarrow y = \frac{4}{9}x - \frac{4}{3} \quad m = \frac{4}{9}$$
$$\perp m = -\frac{9}{4}$$

$$y - (-5) = -\frac{9}{4}(x - 1) \Rightarrow y = -\frac{9}{4}x - \frac{11}{4}$$

4) (3 points each) Find the domain of the following functions:

a)  $f(x) = 6x^3 + 6x^2 - 4x - 4$

$\mathbb{R}$

b)  $g(x) = \frac{x^2 + 9}{x^2 - 9}$

$x^2 - 9 = 0$   
 $x^2 = 9$   
 $x \neq \pm 3$

c)  $h(x) = \frac{-12}{\sqrt{3x-1}}$

$3x - 1 > 0$   
 $x > \frac{1}{3}$

5) (5 points) It was found that the profit  $P$  from selling  $x$  tickets of *The Mythically Nice Professor* can be modeled by the function  $P(x) = -2x^2 + 80x + 15$  where  $P$  is in dollars. Find and interpret the average rate of change from the 8th to the 16th ticket sold.

$$\frac{P(16) - P(8)}{16 - 8} = \frac{783 - 527}{8} = 32$$

Profit is increasing by \$32 per ticket from the 8th to the 16th ticket sold.

6) (3 points each) Consider the following data (source: Census.gov):

| Year   | 2000 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2014 | 2015 |
|--|------|------|------|------|------|------|------|------|------|
| Population of Cleveland, Ohio (in thousands) | 476  | 449  | 442  | 438  | 434  | 431  | 396  | 390  | 386  |

Let  $x$  be the number of years since 2000 and let  $y$  be the population of Cleveland, Ohio (in thousands).

a) Using the LinReg function on your calculator, find the equation of the regression line. Round values to two decimal places:

$$y = -6.36x + 479.19$$

b) Interpret the slope and y-intercept using the language of the problem:

words go here!

c) Predict the population of Cleveland in 2020:

$2020 \Rightarrow x = 20$

$-6.36(20) + 479.19 = 351.99 \text{ thousand}$

d) During what year will there be 250,000 people in Cleveland?

$250 = -6.36x + 479.19 \Rightarrow x = \frac{250 - 479.19}{-6.36} \approx 36.04 + 2000$

During 2036

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7) (2 points each) For the given graph, find the following. Write parts a – d in interval notation. For parts c and d, write in terms of x. For parts e and f, write answer as an ordered pair.

a) The Domain

$$(-\infty, 4]$$

b) The Range

$$[-9, \infty)$$

c) Increases

$$(-4, 2)$$

d) Decreases

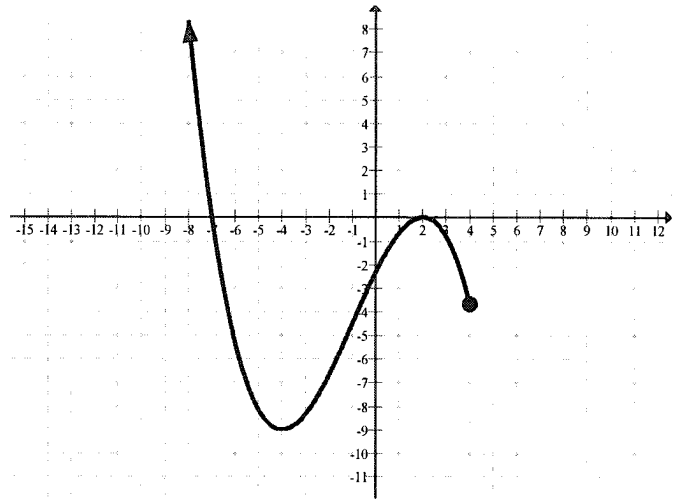
$$(-\infty, -4) \cup (2, 4)$$

e) Relative Maximum(s)

$$(2, 0)$$

f) Relative Minimum(s)

$$(-4, -9)$$



8) (6 points) An evil math instructor wishes to punish students who think that working towards an answer is a good idea. He plans to build 5 adjacent, rectangular pens enclosed on all sides. He has 450 feet of fencing available. He needs to determine a function that will relate the area of the enclosure to the width x; however, he just started playing a video game and he wants you to find this function.



$$6x + 2y = 450 \Rightarrow y = 225 - 3x$$

$$A = xy = x(225 - 3x)$$

$$A(x) = 225x - 3x^2$$

9) (2 points each) For the functions  $f(x) = 3x^2 + 1$  and  $g(x) = \sqrt{5x - 7}$ , find and simplify...

a)  $(f + g)(x)$

$$= 3x^2 + 1 + \sqrt{5x - 7}$$

b)  $(f \circ g)(x)$

$$= 3(\sqrt{5x - 7})^2 + 1$$

$$= 3(5x - 7) + 1$$

$$= 15x - 20$$

c) The domain of  $f \circ g$

$$D_g: x \geq 7/5$$

$$D_f: \mathbb{R}$$

$$D_{f \circ g}: x \geq 7/5$$

10) (3 points) Find two functions  $f$  and  $g$  such that  $H = f \circ g$  where  $H(x) = 8\sqrt{5x^2 + 12} + 7$ :

$$f(x) = 8\sqrt{x} + 7 \quad g(x) = 5x^2 + 12$$

multiple answers.

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11) (6 points) For the function  $f(x) = 4x^2 - 6x + 5$ , find and simplify  $\frac{f(x+h) - f(x)}{h}$ :

$$= \frac{4(x+h)^2 - 6(x+h) + 5 - (4x^2 - 6x + 5)}{h}$$

$$= \frac{4x^2 + 8xh + 4h^2 - 6x - 6h + 5 - 4x^2 + 6x - 5}{h} = \frac{8xh + 4h^2 - 6h}{h}$$

$$= \boxed{8x + 4h - 6}$$

12) (4 points) Determine if the function  $f(x) = \frac{|x|-2}{x^2}$  is even, odd, or neither algebraically:

$$f(-x) = \frac{|-x|-2}{(-x)^2} = \frac{|x|-2}{x^2} \quad \text{Even.}$$

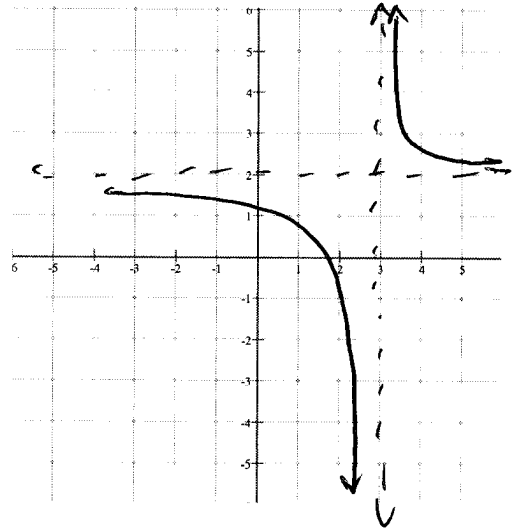
$$|-x| = |x| \quad (-x)^2 = (-x)(-x) = x^2$$

13) (3 points each) For the function  $f(x) = \frac{4}{x-3} + 2$ ...

a) List the steps needed to sketch a graph:

- 1) Right 3
- 2) vertically, stretch by 4
- 3) up 2.

b) Sketch a graph:



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14) (2 points each) Given the point  $(-9, 3)$  on the graph of  $y = f(x)$ , find the **exact value** of the coordinates of the point under the transformation below:

a)  $y = f(x) + 7$

b)  $y = f(x + 1)$

c)  $y = f(-x)$

d)  $y = 3f(x) - 2$

$(-9, 10)$

$(-10, 3)$

$(9, 3)$

$(-9, 7)$

15) (1 point each) Match the following functions with the best description or picture:

G Constant

C Linear

K Identity

B Cube

H Square

J Square root

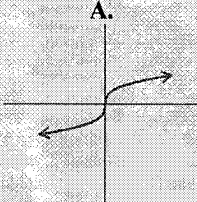

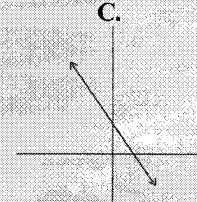
A Cube root

E Greatest-integer

F Reciprocal

D Absolute value

I Piecewise-defined

|  |  |  |
|--|--|--|
| <p><b>A.</b></p>  | <p><b>B.</b></p>  | <p><b>C.</b></p>  |
| <p><b>D.</b><br/><i>The graph is V-shaped</i></p>  | <p><b>H.</b><br/><i>The graph is called a "parabola"</i></p>   | <p><b>I.</b><br/><i>Made up of other functions</i></p>   |
| <p><b>E.</b><br/><i>Also called the step function</i></p>  | <p><b>J.</b><br/><i>The graph is half of a parabola</i></p>  | <p><b>K.</b><br/><i>Bisects the first and third quadrant</i></p>                                       |
| <p><b>F.</b><br/><i>The domain and range do not include zero</i></p>                                 |  |  |
| <p><b>G.</b><br/><i>The range is one number</i></p>  |  |  |

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