

1) (3 points each) Find the inverse of the following functions:

a) $\{(8,4), (0,7), (12,9), (6,6)\}$

$\{(4,8), (7,0), (9,12), (6,6)\}$

b) $f(x) = \frac{2}{x-3}$

① $y = \frac{2}{x-3}$

② $x = \frac{2}{y-3}$

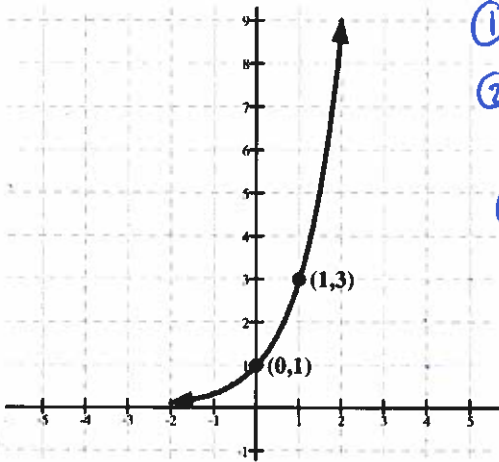
③ $xy - 3x = 2$

$xy = 2 + 3x$

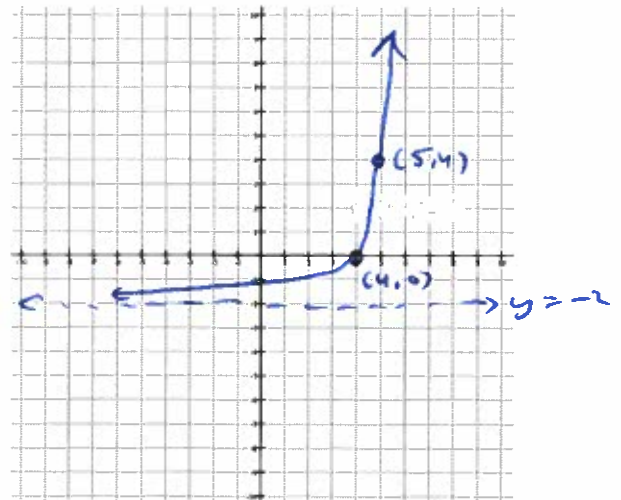
$y = \frac{2+3x}{x}$

④ $f^{-1}(x) = \frac{2+3x}{x}$

2) (5 points) Graph $g(x) = 2 \cdot 3^{x-4} - 2$ by transforming the given function $y = 3^x$. Be sure to move and label the given points and asymptotes.



- ① right 4
- ② vert. stretch by 2
- ③ down 2



3) (4 points) Find the inverse of the function $g(x) = 2 \cdot 3^{x+4} - 2$.

① $y = 2 \cdot 3^{x+4} - 2$

② $x = 2 \cdot 3^{y+4} - 2$

③ $x+2 = 2 \cdot 3^{y+4}$

$\frac{x+2}{2} = 3^{y+4}$

$\log_3\left(\frac{x+2}{2}\right) = y+4$

$y = \log_3\left(\frac{x+2}{2}\right) - 4$

④ $g^{-1}(x) = \log_3\left(\frac{x+2}{2}\right) - 4$

4) (3 points) Given that $\log_a 9 = 2.566$ and $\log_a 4 = 1.555$, find the value for $\log_a \frac{16}{9}$:

$\log_a \frac{16}{9} = \log_a \frac{4^2}{9} = \log_a 4^2 - \log_a 9 = 2 \log_a 4 - \log_a 9$

$= 2(1.555) - 2.566 = 0.544$

5) (2 points each) Finish the explanation of the given equations. **Do not solve the equations.**

a) $5^{4x-7} = 125$ is a one-to-one exponential equation because...

word (two reasons)

b) $\log_{17}(2x+8) = 16$ is not a one-to-one logarithmic equation because...

more words (two reasons)

6) (5 points each) Solve for the variable. Be sure to find the exact value.

a) $8^{2x-1} = 16^{x+4}$

$$(2^3)^{2x-1} = (2^4)^{x+4}$$

$$2^{6x-3} = 2^{4x+16}$$

$$6x-3 = 4x+16$$

$$2x = 19$$

$$x = \frac{19}{2}$$

b) $10e^{4x+1} = 13$

$$e^{4x+1} = \frac{13}{10}$$

$$\ln \frac{13}{10} = 4x+1$$

$$x = \frac{\ln \frac{13}{10} - 1}{4}$$

c) $\log_2(x+1) - \log_2(x+2) = \log_2 8$

$$\log_2 \left(\frac{x+1}{x+2} \right) = \log_2 8$$

$$\frac{x+1}{x+2} = 8$$

$$x+1 = 8x+16$$

$$-15 = 7x$$

$$x = -\frac{15}{7}$$

$$\emptyset$$

d) $\ln(3x+4) + 3 = 12$

$$\ln(3x+4) = 9$$

$$e^9 = 3x+4$$

$$\frac{e^9 - 4}{3} = x$$

7) (3 points each) After drinking six espressos in one sitting, Mike's heartrate grew exponentially without bound. Using $P(t) = P_0 e^{kt}$ where P is the heartrate in beats per minute and t is the number of minutes after he finished his 6th espresso...

a) Determine the **exact value** for the growth rate k if his initial heart rate was 64 beats per minute but grew to 129 beats per minute after 5 minutes.

$$129 = 64 e^{k \cdot 5}$$

$$\frac{129}{64} = e^{5k}$$

$$\ln \frac{129}{64} = 5k$$

$$k = \frac{\ln \frac{129}{64}}{5}$$

b) Using the exact value of k from part a, determine Mike's heartrate after 10 minutes. Round to the nearest whole number:

$$P(10) = 64 e^{\frac{\ln \frac{129}{64}}{5} \cdot 10}$$

$$\approx 260 \text{ bpm}$$



Mike was able to see a hummingbird slowly flap its wings around the 4th espresso.

8) (4 points each) Find the first four terms of the following sequences using fractions as necessary. Determine if they are arithmetic, geometric, or neither. If it is arithmetic, determine the common difference. If it is geometric, determine the common ratio.

a) $\{5n - 2\}$

$n=1$	$5(1)-2$	3
$n=2$	$5(2)-2$	8
$n=3$	$5(3)-2$	13
$n=4$	$5(4)-2$	18
$n=5$	$5(5)-2$	23

arithmetic $d=5$

b) $a_1 = 7, a_2 = 2, a_{n+2} = 2a_n - a_{n+1}, n \geq 1$

$$a_1 = 7 \quad a_2 = 2$$

$$a_3 = 2(a_1) - a_2 = 12$$

$$a_4 = 2(a_2) - a_3 = -8$$

$$a_5 = 2(a_3) - a_4 = 32$$

$$\{7, 2, 12, -8, 32\}$$

Neither

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9) (4 points each) Given the sequence below, find the general term a_n . For part a only, use the formula $a_n = a_1 + (n - 1)d$ and simplify:

a) 8, 5, 2, -1, ...

$a_1 = 8 \quad d = -3$

b) $\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{3}}{2}, \frac{2}{\sqrt{5}}, \frac{\sqrt{5}}{\sqrt{6}}, \dots$

$a_n = \frac{\sqrt{n}}{\sqrt{n+1}}$

$a_n = 8 + (n-1)(-3)$
 $a_n = -3n + 11$

10) (3 points) Find the sum below. Be sure to write out the individual terms:

$\sum_{k=6}^{10} (-1)^k \cdot k^2 = (-1)^6 \cdot 6^2 + (-1)^7 \cdot 7^2 + (-1)^8 \cdot 8^2 + (-1)^9 \cdot 9^2 + (-1)^{10} \cdot 10^2$
 $= 70$

11) (3 points each) Write the follow series in sigma notation. Refer to problem 9:

a) $8 + 5 + 2 + (-1) + \dots + (-58)$

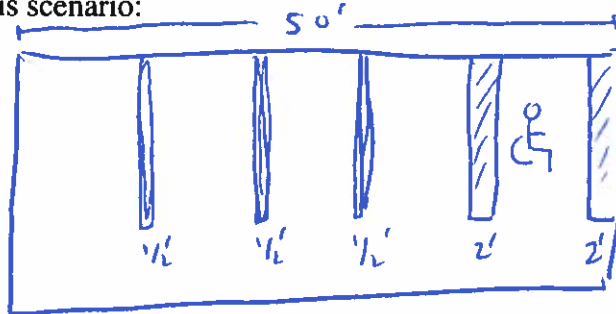
$\sum_{k=1}^{23} (-3k + 11)$
 $-58 = -3k + 11 \Rightarrow k = 23$
 $-64 = -7k \Rightarrow k = 23$

b) $\frac{1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{2} + \frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{\sqrt{6}} + \dots + \frac{10}{\sqrt{101}}$

$\sum_{k=1}^{100} \frac{\sqrt{k}}{\sqrt{k+1}}$

12) (3 points each) A 50-foot long parking lot is to be painted so that it contains 5 equal-width parking spots. The right-most spot is designated for handicap only and will be flanked by a 2-foot wide line on each side to allow for the doors to open fully. The other 4 spots will be separated by a 1/2 foot wide line. There is no line drawn at the left edge of the parking lot.

a) Draw a picture for this scenario:



b) How wide is each of the 5 parking spots?

$\frac{50 - \frac{1}{2} \cdot 3 - 2 \cdot 2}{5} = 8.9 \text{ ft}$

c) Starting your count from the left-most edge of the parking lot, determine the location of the left-edge of the 1/2 foot wide lines:

$8.9', 18.3', 27.7'$
 $\uparrow \quad \quad \quad \uparrow$
 $+\frac{1}{2} + 8.9 \quad +\frac{1}{2} + 8.9$

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13) (4 points each) Evaluate the sums using either formula: $S_n = \frac{n}{2}(2a_1 + (n-1)d)$ or $S_n = \frac{n}{2}(a_1 + a_n)$. Write answers as improper fractions as needed:

a) $\sum_{k=1}^{500} (10k + 2) = \frac{500}{2}(12 + 5002)$
 $n = 500$
 $a_1 = 10(1) + 2 = 12$
 $a_{500} = 10(500) + 2 = 5002$
1,253,500

b) $\sum_{k=50}^{62} \frac{2k+5}{4} = \frac{13}{2} \left(\frac{105}{4} + \frac{126}{4} \right) = \frac{1521}{4}$
 $n = 62 - 50 + 1 = 13$
 $a_1 = \frac{2(50) + 5}{4} = \frac{105}{4}$
 $a_{13} = \frac{2(62) + 5}{4} = \frac{126}{4}$

14) (2 points) Short answer: Why can't you find an infinite sum of terms of an arithmetic sequence, but you can under certain constrictions of a geometrics sequence.

Converge...

15) (4 points) Find the sum $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots + \frac{128}{2187}$ using $S_n = a_1 \frac{1-r^n}{1-r}$. Write answer as an improper fraction using MATH > FRAC on your calculator:

$a_n = 1 \left(\frac{2}{3}\right)^{n-1}$ $\left(\frac{2}{3}\right)^{n-1} = \frac{128}{2187} \Rightarrow \left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^7 \Rightarrow n-1=7$
 $a_1 = 1$ $r = \frac{2}{3}$ $n = 8$
 $S_8 = 1 \cdot \frac{1 - \left(\frac{2}{3}\right)^8}{1 - \frac{2}{3}} = \frac{6305}{2187}$

16) (4 points) Gilberto's daughter tosses the pony she's always wanted as a child off of a 240 foot cliff. Thankfully, the pony is made of rubber and bounces gleefully, rebounding $\frac{4}{5}$ of the distance fallen. How far vertically does the pony travel? Use the formula $S_\infty = \frac{a_1}{1-r}$ in your answer.



$\frac{\text{drop } 240}{1 - 4/5} + \frac{\text{rebound } 192}{1 - 4/5}$

= 2160 ft

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