1) (2 points) Solve for the variable in $x^4 - 1 = 0$. *Hint, there are four answers.*

$(x^2+1)(x^2-1) = 0$

$x^2 = -1 \qquad x^2 = 1$

$\boxed{x = \pm i \qquad x = \pm 1}$

2) (2 points each) For the function $f(x) = 2x^2 - 8x + 1$, determine…

a) If it opens up or down. How do you know?

up $a = 2 > 0$

b) The coordinates of the vertex:

$x = -\frac{b}{2a} = \frac{-(-8)}{2(2)} = 2$

$f(2) = -7$

$(2, -7)$

c) The domain:

$\mathbb{R}$

d) The range:

$[-7, \infty)$

e) Interval of increase:

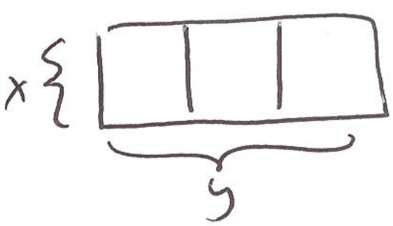
$(2, \infty)$

f) Interval of decrease:

$(-\infty, 2)$

3) (5 points) Paul is a spice worm farmer and he wants to create 3 adjacent rectangular pens that which are enclosed on all sides using 200 feet of fence for his creatures. What should the dimensions of the enclosure be to maximize area? Also, what is the maximum area? Be sure to draw a picture for this scenario.

Here is one of Paul's spice worms waiting for its morning truffle snack.



$4x + 2y = 200$

$\Rightarrow y = 100 - 2x$

$A = xy$

$A(x) = x(100 - 2x) = -2x^2 + 100x$

$x = \frac{-100}{2(-2)} = 25$ ft $\qquad y = 100 - 2(25) = 50$ ft $\qquad A(25) = 1250$ ft²

4) (2 points each) Solve for the variable. $\frac{3x-7}{x^2-9} - \frac{5}{x-3} = \frac{7}{x+3}$

multiply everything by $(x+3)(x-3)$

$3x - 7 - 5(x+3) = 7(x-3)$

$3x - 7 - 5x - 15 = 7x - 21$

$-1 = 9x \Rightarrow \boxed{x = -\frac{1}{9}}$

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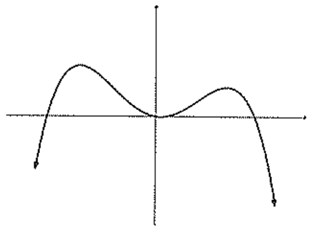
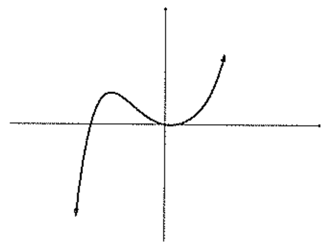
5) (2 points each) Give an example of a function which will have similar arrowheads to the function below:

a)



b)



leading term requirement

$f(x) =$ neg $\cdot\, x^{\text{even}}$

$f(x) =$ pos $\cdot\, x^{\text{odd}}$

6) (3 points each) For the function $f(x) = (x+4)(x-2)^2(x-5)\ldots$

a) Find the leading term and state which quadrants the arrowheads will be in:

$x^{4 \to \text{even}}$

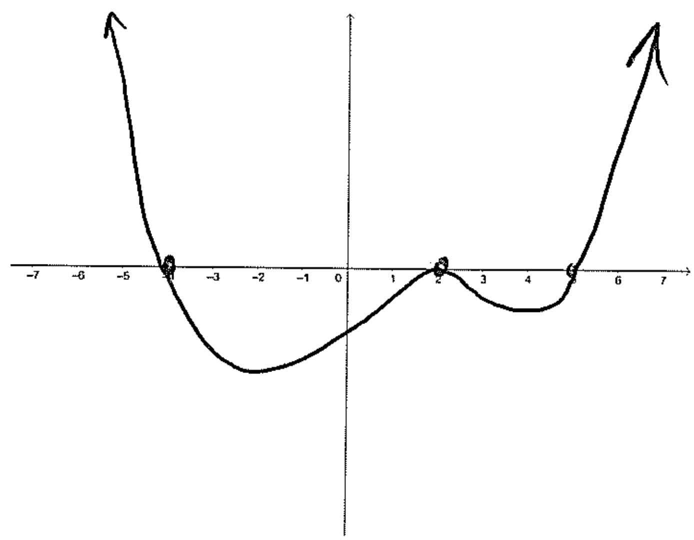
pos

Q I, II

b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
-4	1	c
2	2	t
5	1	c

c) Sketch the graph based on parts *a* and *b*:



7) (3 points each) For the function $f(x) = -3x^4 + 12x^2\ldots = -3x^2(x^2-4) = -3x^2(x+2)(x-2)$

a) Find the leading term and state which quadrants the arrowheads will be in:

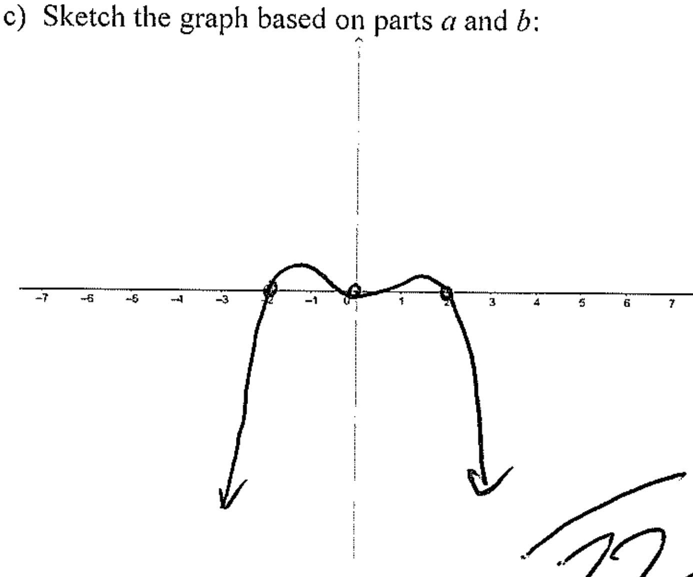
$-3x^{4 \to \text{even}}$

neg   Q III, IV

b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
-2	1	C
0	2	T
2	1	C

c) Sketch the graph based on parts *a* and *b*:



8) (2 points each) Form a polynomial function of degree four that meets the following requirements. **Be sure to leave your answer in factored form:**

a) Has zeros at 8, 3, 5, and 0:

or x

$y=(x-8)(x-3)(x-5)(x-0)$

b) Has the same zeros and multiplicity as in part *a* but is a different function:

$y=5(x-8)(x-3)(x-5)x$

any non-zero #

c) Has a zero at $2+3i$, and 8 is a zero of multiplicity 2:

$y=(x-(2+3i))(x-(2-3i))(x-8)^2$

9) (3 pts *a;* 2 pts others) Consider the functions $f(x)=6x^3+x^2-12x+5$ and $g(x)=3x^2+2x-5$.

a) Divide $f(x)$ by $g(x)$ using long division:

$$
\begin{array}{r}
2x \quad -1 \\
3x^2+2x-5 \,\overline{)\, 6x^3+x^2-12x+5} \\
-(6x^3+4x^2-10x) \\
\hline
-3x^2-2x+5 \\
-(-3x^2-2x+5) \\
\hline
0
\end{array}
$$

b) Based on your work in part *a*, was $g(x)$ a factor of $f(x)$? Why or why not?

yes

Remainder $=0$

c) What is the equation of the oblique asymptote of the rational function $y=\dfrac{6x^3+x^2-12x+5}{3x^2+2x-5}$?

$y=2x-1$

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10) (7 points each) Factor the polynomial completely by first listing the possible rational roots and then using synthetic division and your calculator.

a) $f(x)=x^3-5x^2-12x+36$

$p=\pm 1,2,3,4,6,9,12,18,36$

$q=\pm 1 \quad \frac{p}{q}=p$

$$\begin{array}{r|rrrr} -3 & 1 & -5 & -12 & 36 \\ & & -3 & 24 & -36 \\ \hline & 1 & -8 & 12 & \boxed{0} \end{array}$$

$(x+3)(x^2-8x+12)$

$\boxed{(x+3)(x-2)(x-6)}$

b) $g(x)=x^4-2x^3-10x^2+16x+40$

$p=\pm 1,2,4,5,8,10,20,40$

$q=\pm 1 \quad \frac{p}{q}=p$

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -10 & 16 & 40 \\ & & -2 & 8 & 4 & -40 \\ \hline -2 & 1 & -4 & -2 & 20 & \boxed{0} \\ & & -2 & 12 & -20 & \\ \hline & 1 & -6 & 10 & \boxed{0} & \end{array}$$

equivalently!

$(x+2)^2 \cdot$

$\boxed{(x+2)^2(x^2-6x+10)(x-(3+2i))(x-(3-2i))}$

$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}=\frac{6\pm\sqrt{-4}}{2}=3\pm 2i$

11) (3 points each) For the function $f(x)=\frac{x-2}{x^2-x-2}$, find...

a) The domain:

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x\neq 2,-1$

b) The x- and y-intercepts:

$\underline{x\text{-int}}$

$x=2$

ϕ

not in domain

$\underline{y\text{-int}}$

$f(0)=\frac{-2}{-2}=1$

$(0,1)$

c) Any vertical asymptotes and holes:

$\underline{x=-1}$: $-1-2\neq 0$ $\quad x=-1$ VA

$\underline{x=2}$: $2-2=0$ Hole

$\frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}=\frac{1}{x+1}$ @ $x=2$

$\frac{1}{2+1}=\frac{1}{3}$

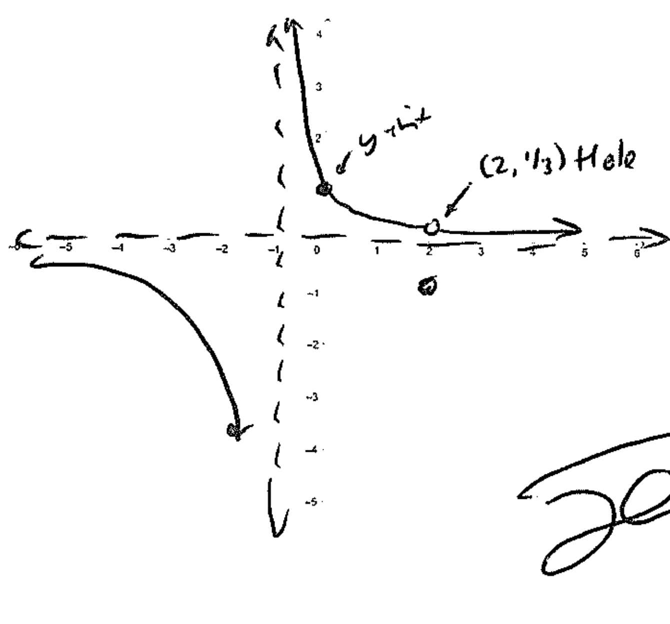
$(2, 1/3)$ Hole

d) Any horizontal or oblique asymptotes:

$y=0$
HA

e) Sketch a graph based on the above.

Hint: you should see a transformation of $y=\frac{1}{x}$ *in your work. Use that!*



12) (2 points each) Fill in the blank:

a) If c is a zero of a function f, then $f(c) =$ words, and word is a factor.

b) Numbers not in the domain of a rational function lead to words.

13) (3 points each) Short answer. Clearly explain how to find the following algebraically:

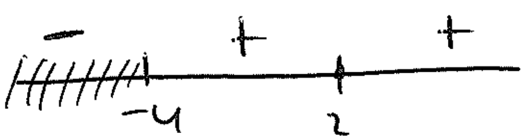
a) Vertical Asymptotes and Holes:

b) Horizontal and Oblique Asymptotes:

moar
words

14) (3 points each) Solve for the variable. Write answer in interval notation:

$$(x+4)(x-2)^2 \le 0$$



$(-\infty, -4] \cup \{2\}$

↳ oops accidentally evil.

Extra Credit (2 points) :

Find the equation of a rational <u>function</u> **in factored form** that has the following properties:

a) Hole at $x = 9$

b) Vertical Asymptotes at $x = -3$ and $x = 5$

c) x-intercepts at $x = \frac{1}{7}$ and $x = -6$

d) Horizontal asymptote at $y = 7$

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