1) (6 points each) Solve the system using the methods listed below. Write answer as an ordered triple.

a) The Elimination method:

$$\begin{cases} 2x + y = -6 \\ x - 7y = -33 \end{cases} \quad x(-2)$$

$2x + y = -6$

$-2x + 14y = 66$

$15y = 60$

$y = 4$

$x - 7(4) = -33$

$x = -5$

$(-5, 4)$

b) Gauss-Jordan method:

$$\left[\begin{array}{cc|c} 2 & 1 & -6 \\ 1 & -7 & -33 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & -7 & -33 \\ 2 & 1 & -6 \end{array}\right] \xrightarrow{-2R_1 + R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -7 & -33 \\ 0 & 15 & 60 \end{array}\right]$$

$$\xrightarrow[7R_2 + R_1 \to R_1]{\frac{1}{15}R_2 \to R_2} \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 4 \end{array}\right]$$

$(-5, 4)$

2) (1 point) Verify that you made absolutely sure that your answer to 1a is the same as in 1b by signing your name here My name is Bob B.J.______. You will not receive the credit if the work does not support the same answer.

3) For the following problem:

A person invested \$4,200 for one year, part at 8%, part at 10%, and the remainder at 12%. The total annual return was \$716. The total amount of money invested in the 12% was \$300 more than the amounts invested at 8% and 10% combined. How much was invested at each rate?

a) (3 points) Name and define your variables for this problem:

x = amt invested at 8%

y = " " " 10%

z = " " " 12%

b) (4 points) Set up **BUT DO NOT SOLVE** a system of equations for this problem:

$$\begin{cases} x + y + z = 4200 \\ 0.08x + 0.10y + 0.12z = 716 \\ z = 300 + x + y \xrightarrow{\ ''\ } -x - y + z = 300 \end{cases}$$

4) (2 points each) What property must be true to...

a) Add or subtract matrices?

Everybody

b) Multiply matrices?

Dance!

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5) (4 points each) From Monday – Friday, you like to spend 2 hours playing video games, 1 hour reading, and 3 hours doing math homework each day. On Saturday and Sunday, you like to spend 4 hours playing video games, no time reading, and 6 hours doing math homework each day.

a) Create two 1×3 matrices, called M and S respectively, where the first shows the amount of time spent on the three activities on Monday **only** and the other shows the amount of time spent on the three activities on Saturday **only**. Be sure to label the rows and columns.

$$M = \begin{array}{c} \\ hr \end{array}\begin{array}{c} \begin{array}{ccc} Vg & read & mh \end{array} \\ \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \end{array} \qquad S = \begin{array}{c} \\ hr \end{array}\begin{array}{c} \begin{array}{ccc} Vg & read & mh \end{array} \\ \begin{bmatrix} 4 & 0 & 6 \end{bmatrix} \end{array}$$

b) Using scalar multiplication and addition, find the total time spent on the three activities for the entire week. Be sure to label the rows and columns.

$$5M+2S = \begin{array}{c} \\ hr \end{array}\begin{array}{c} \begin{array}{ccc} Vg & read & mh \end{array} \\ \begin{bmatrix} 18 & 5 & 27 \end{bmatrix} \end{array}$$

6) (4 points each) Brothers Romulus and Remus do chores at home to earn an allowance. They plan to establish the city of Rome with all of their earnings. Romulus will do yardwork 2 times a week, wash the dishes 3 times a week, walk the dog 4 times a week, and vacuum 1 time a week. Remus will do yardwork 1 time a week, wash the dishes 4 times a week, walk the dog 2 times a week, and vacuum 3 time a week. They are paid \$5 every time they do yardwork, \$2 every time they wash the dishes, \$7 every time they walk the dog, and \$4 every time they vacuum.

a) Create a 2×4 matrix called B showing the names of the brothers and the number of times they do the given chores. Create a 4×1 matrix called A showing the amount paid for each chore. Be sure to label the rows and columns.

$$B = \begin{array}{c} \\ Romulus \\ Remus \end{array}\begin{array}{c} \begin{array}{cccc} yard & dish & dog & vacuum \end{array} \\ \begin{bmatrix} 2 & 3 & 4 & 1 \\ 1 & 4 & 2 & 3 \end{bmatrix} \end{array} \qquad A = \begin{array}{c} \\ yard \\ dish \\ dog \\ vacuum \end{array}\begin{array}{c} \$ \\ \begin{bmatrix} 5 \\ 2 \\ 7 \\ 4 \end{bmatrix} \end{array}$$

b) Find the product BA and interpret each value. Be sure to label the rows and columns.

$$BA = \begin{array}{c} \\ Romulus \\ Remus \end{array}\begin{array}{c} \$ \\ \begin{bmatrix} 48 \\ 39 \end{bmatrix} \end{array}$$

money earned per week for each brother.

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7) (6 points part a; 3 points part b) For the system $\begin{cases} x-2y=-2 \\ 2x-3y=-1 \end{cases}$...

a) Find the inverse of the coefficient matrix algebraically using the Gauss-Jordan Method:

$$\left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 2 & -3 & 0 & 1 \end{array}\right] \xrightarrow{-2R_1+R_2 \to R_2} \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$2R_2+R_1 \to R_1 \quad \left[\begin{array}{cc|cc} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\underbrace{\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}}_{\text{Inverse}}$$

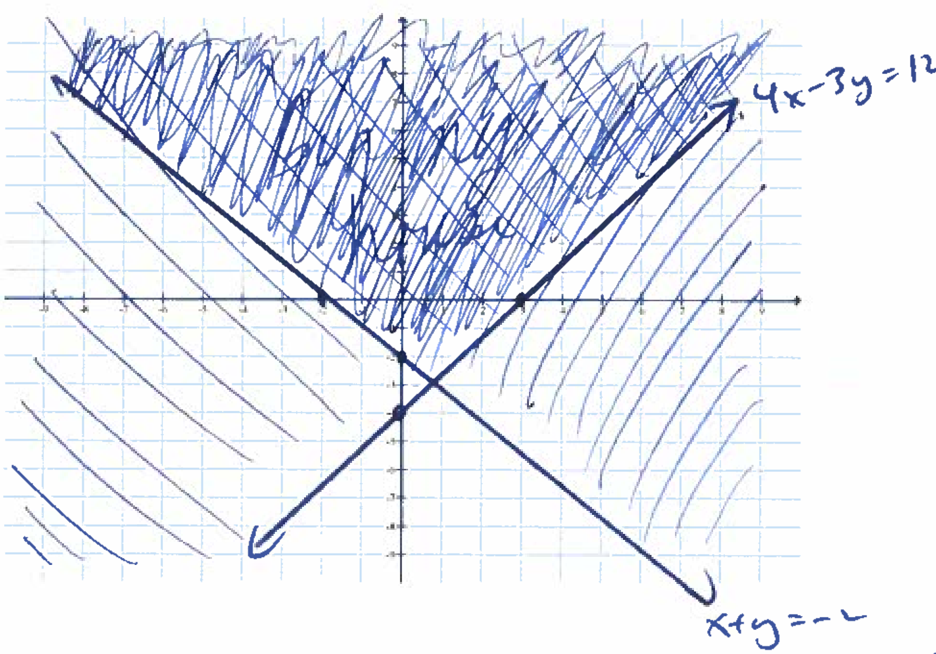
b) Solve the system using the matrix inverse from part a. Write answer as an ordered pair.

$$\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \quad (4,3)$$

8) (6 points each) For the following system of equations, sketch the feasible region and determine if the feasible region is bounded or unbounded. Be sure to shade your answer in the darkest:

$\begin{cases} 4x-3y \le 12 \\ x+y \ge -2 \end{cases}$

Unbounded



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9) Consider the following:

A desk company produces two models of desks: the Sirrus and Achenar. The Sirrus model takes 3 hours to assemble, ½ hour to stain, and 2 hours to detail. The Achenar model takes 2 hours to assemble, 1 hour to stain, and 2 hours to detail. The maximum number of hours available to assemble is 24 per day, the maximum number of hours available to stain is 8 hours, and the maximum number of hours available to detail is 18 per day. The company earns a profit of $120 per Sirrus model and $160 per Achenar model. Find the number of models produced per day in order to maximize profit.

a) (4 points) Name and define the variables for this problem. Also write the associated linear programming problem.

x = # of Sirrus desks

y = # of Achenar desks

Maximize $z = 120x + 160y$

subject to

assemble $3x + 2y \le 24$

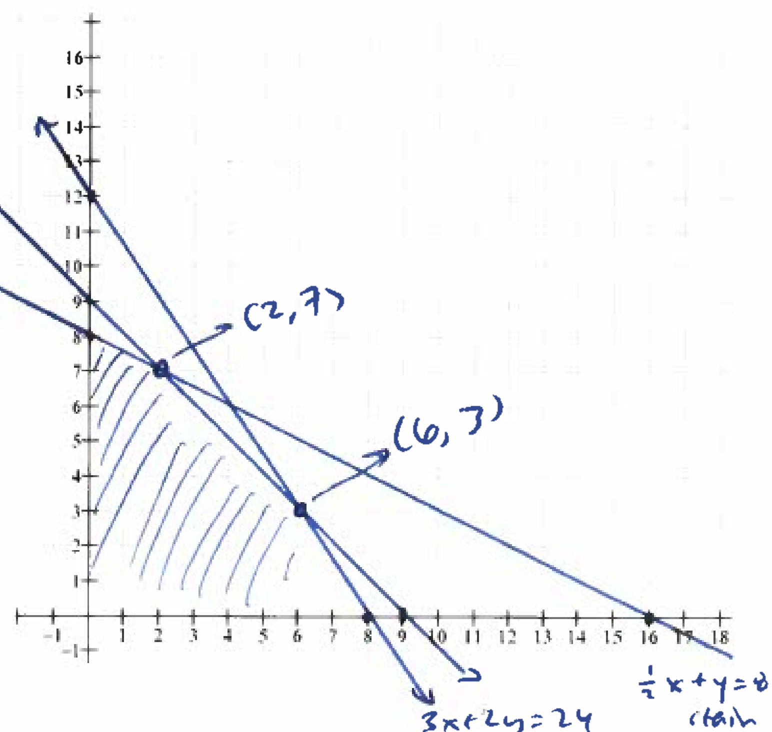
stain $\frac{1}{2}x + y \le 8$

detail $2x + 2y \le 18$

$x, y \ge 0$

b) (8 points) Solve the problem using the Corner Point Method. Be sure to interpret your answer.

(x,y)	z = 120x+160y
(0,0)	0
(8,0)	960
(0,8)	1280
(2,7)	1360
(6,3)	1200



They should make 2 Sirrus & 7 Achenar desks for a profit of $1360.

c) (3 points) Determine how many (if any) hours to assemble, stain, and detail are left over.

Assemble: $24 - (3\cdot 2 + 2\cdot 7) = 4$ hrs

Stain: $8 - (\frac{1}{2}\cdot 2 + 7) = 0$ hrs

detail: $18 - (2\cdot 2 + 2\cdot 7) = 0$ hrs

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10) (2 points each) Consider a Maximization LP in standard form…

a) How is Pivot Column determined?

b) Based on your answer from part *a*, why is the Pivot Column picked this way?

c) What does the Pivot Column tell us geometrically?

d) How is the Pivot Row determined?

e) Based on your answer to part *d*, why is the Pivot Row picked this way?

f) What does the Pivot Row tell us geometrically?

11) (8 points) For the tableau below:

i) Fill in the Basic Variable column.

ii) Write the corresponding augmented coordinates (x_1, x_2, s_1, s_2) and objective function value. Is it a corner point? Why or why not?

iii) Circle the pivot element.

iv) Declare what the new Basic Variable and Non-basic Variable will be *after* the pivot. **Do not actually pivot**

BV	x_1	x_2	s_1	s_2	z	RHS
X_1	1	4	0	6	0	8
S_1	0	6	1	5	0	10
z	0	-12	0	-6	1	27

Augmented coordinates: (8, 0, 10, 0) Obj. function value: 27

New BV: X_2
New Non-BV: S_1

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12) (10 points) Solve the linear programming problem below using the Simplex Method. Be sure to show your initial simplex tableau and the resulting tableau after each pivot. Declare the optimal solution and value on the lines below.

Maximize $z = 120x_1 + 160x_2$

subject to

$$3x_1 + 2x_2 \le 24$$

$$\frac{1}{2}x_1 + x_2 \le 8$$

$$2x_1 + 2x_2 \le 18$$

$$x_1, x_2 \ge 0$$

$$\begin{array}{c} BV \\ s_1 \\ s_2 \\ s_3 \\ z \end{array} \begin{array}{c} \begin{array}{ccccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & z & RHS \end{array} \\ \left[\begin{array}{ccccccc|c} 3 & 2 & 1 & 0 & 0 & 0 & 24 \\ 1/2 & 1 & 0 & 1 & 0 & 0 & 8 \\ 2 & 2 & 0 & 0 & 1 & 0 & 18 \\ \hline -120 & -160 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

$-2R_2 + R_1 \to R_1$

$-2R_2 + R_3 \to R_3$

$160R_2 + R_4 \to R_4$

$$\begin{array}{c} BV \\ s_1 \\ x_2 \\ s_3 \\ z \end{array} \begin{array}{c} \begin{array}{ccccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & z & RHS \end{array} \\ \left[\begin{array}{ccccccc|c} 2 & 0 & 1 & -2 & 0 & 0 & 8 \\ 1/2 & 1 & 0 & 1 & 0 & 0 & 8 \\ 1 & 0 & 0 & -2 & 1 & 0 & 2 \\ \hline -40 & 0 & 0 & 160 & 0 & 1 & 1280 \end{array}\right] \end{array}$$

$-2R_3 + R_1 \to R_1$

$-\frac{1}{2}R_3 + R_2 \to R_2$

$40R_3 + R_4 \to R_4$

$$\begin{array}{c} BV \\ s_1 \\ x_2 \\ x_1 \\ z \end{array} \begin{array}{c} \begin{array}{ccccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & z & RHS \end{array} \\ \left[\begin{array}{ccccccc|c} 0 & 0 & 1 & 2 & -2 & 0 & 4 \\ 0 & 1 & 0 & 2 & -1/2 & 0 & 7 \\ 1 & 0 & 0 & -2 & 1 & 0 & 2 \\ \hline 0 & 0 & 0 & 80 & 40 & 1 & 1360 \end{array}\right] \end{array}$$

Optimal solution: $x_1 = 2, x_2 = 7$ Optimal value: 1360

Seems... familiar...

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