1) (3 points) Write as one logarithm $9\log_4 x + 5\log_4 y - 10\log_4 z$:

power $= \log_4 x^9 + \log_4 y^5 - \log_4 z^{10}$ product $= \log_4(x^9 y^5) - \log_4 z^{10}$

Quotient $= \log_4 \frac{x^9 y^5}{z^{10}}$

2) (4 points each) Solve for the variable. **Be sure to find the exact value.**

a) $8^{2x-1} = 32^{x+4}$

$(2^3)^{2x-1} = (2^5)^{x+4}$

$2^{6x-3} = 2^{5x+20}$

$6x-3 = 5x+20 \Rightarrow x = 23$

b) $12e^{4x+1} = 13$

$e^{4x+1} = \frac{13}{12}$

$\ln\frac{13}{12} = 4x+1 \Rightarrow \frac{\ln\frac{13}{12} - 1}{4} = x$

c) $\log_2(x+1) - \log_2(x+2) = \log_2 8$

$\log_2\left(\frac{x+1}{x+2}\right) = \log_2 8$

$\frac{x+1}{x+2} = 8 \Rightarrow x+1 = 8x+16$

$-15 = 7x$

$x = -\frac{15}{7}$ $\emptyset$

d) $\ln(3x+4) + 9 = 12$

$\ln(3x+4) = 3$

$e^3 = 3x+4 \Rightarrow x = \frac{e^3 - 4}{3}$

3) (4 points) After drinking six espressos in one sitting, Mike's heartrate grew exponentially without bound. Using $f(t) = y_0 e^{kt}$ where f is the heartrate in beats per minute and t is the number of minutes after he finished his 6th espresso…

Mike was able to see a hummingbird slowly flap its wings around the 4th espresso.

a) Determine the exact value for the growth rate k if his initial heart rate was 63 beats per minute but grew to 127 beats per minute after 3 minutes.

$127 = 63e^{k\cdot 3}$

$\frac{127}{63} = e^{3k}$

$\ln\frac{127}{63} = 3k$

$k = \frac{\ln\frac{127}{63}}{3}$

b) Using the exact value of k from part a, determine Mike's heartrate after 10 minutes. Round to the nearest whole number:

$f(10) = 63e^{\frac{\ln\frac{127}{63}}{3}\cdot 10}$

$\approx$ 652 bpm

4) (4 points) Frank invests in the Darko Bank that offers a 1.25% simple interest rate. He invests $550 at this rate for 16 years. How much is in the account after that time and how much interest was earned?

Simple interest

total: $A = 550(1 + 0.0125\cdot 16) =$ \$660

Interest $660 - 550 =$ \$110

5) (4 points) You deposit $2,700 into the account as a lump sum. The account carries a 1.05% interest rate compounded **quarterly**. How much money will be in the account after 20 years and how much interest did he earn?

Compound interest

$A = 2700\left(1+\frac{0.0105}{4}\right)^{4\cdot 20} \approx \$3330.01$ total

interest: $3330.01 - 2700 = \$630.01$

6) (4 points) How many years would it take $5,000 to double in an account that has a 1.7% annual interest rate compounded monthly? Be sure to solve the problem algebraically. *Hint: You need to solve for something in the exponent.*

Compound interest

$10000 = 5000\left(1+\frac{0.017}{12}\right)^{12t}$

$2 = \left(1+\frac{0.017}{12}\right)^{12t}$

$\log_{1+\frac{0.017}{12}}(2) = 12t$

$t = \frac{\log_{1+\frac{0.017}{12}}(2)}{12} \approx 40.80$ yrs

7) (4 points) How much should be invested now so that in 15 years there will be $7,800 in an account that offers a 2.35% annual interest rate compounded quarterly?

present value

$P = 7800\left(1+\frac{0.0235}{4}\right)^{-4\cdot 15} \approx \$5488.50$

8) (4 points) Which is a better way to invest? Option A: 6.3% compounded semi-annually or Option B: 6.25% compounded monthly? **Write answer as a percent rounded to two decimal places.**

APY

A: $\left(1+\frac{0.063}{2}\right)^{2}-1 \rightarrow 6.40\%$

B: $\left(1+\frac{0.0625}{12}\right)^{12}-1 \rightarrow 6.43\%$

B better!

9) (4 points) Fill in the chart with the appropriate **name** of the formula. Assume that this chart is used for those formulas related to multiple deposits/payments:

	Working Years	Retirement Years
Know the Payment	bacon	pancakes
Do Not Know the Payment	makin' bacon	pancakes

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10) (3 points) What is the **major theoretical** distinction between Compound Interest and Future Value of an Annuity?

take some bacon and I'll put it in a pancake

11) (2 points each) Consider the problem below. For each part, only determine the formula that is needed to answer the question **but do not find the value. Just write the name of the formula and explain why**.

For the first 10 years of Holly's life, her parents were able to deposit $275 a month into an account that offered a 8.25% annual interest rate compounded monthly. After that time, the parents could no longer contribute to the account and just allowed the balance to sit there for the next 8 years at the same interest rate and rate of compounding.

a) Which formula would be needed to determine the amount of money in the account after the first 10 years? Explain why.

Future Value because..

b) Which formula would be needed to determine the amount of money in the account after the ~~18~~ 10 years? Explain why.

Compound Interest

because...

c) Now after those 18 years, Holly wishes to take equal withdraws from the account at the same interest rate over the next 5 years until the account has a balance of $0. Which formula would be needed and why?

Amortization because...

work
40 yrs
6.25%
retire
$5700
30 yrs
6.25%
25% tax

12) (5 points each) Schmidt works out that he would need $5,700 a month during his retired years. He is currently 25 years old and plans to work until his is 65. He assumes that he would need to make withdraws for 30 years past his retirement and that he's in a 25% tax bracket. Assuming he finds an account that will offer him a 6.25% annual interest rate compounded monthly for the entire duration of the account…

a) How much should he have in his account at retirement?

PVoA

$$PV = \frac{5700\left(1-\left(1+\frac{0.0625}{12}\right)^{-12\cdot 30}\right)}{\left(\frac{0.0625}{12}\right)}$$

$= \$925,749.68$ → After tax

keep 75% ÷ 0.75 = $1,234,332.91

b) How much should he deposit monthly during his working years to ensure he meets his goal?

Sinking Fund

$$PMT = \frac{1,234,332.91\left(\frac{0.0625}{12}\right)}{\left(\left(1+\frac{0.0625}{12}\right)^{12\cdot 40}-1\right)}$$

$= \$578.98$

19

work	retire
$525/mo	?
7.25%	7.25%
30 yr	25 yr

13) Balthier is 30 years old and is working as a local sky pirate. He is able to deposit $525 a month into a Pirate Bank 401-k which offers a 7.25% annual interest rate. He does this for 30 years. After that time, he will retire. He wishes, over the next 25 years, to take out equal withdraws until the account is emptied. Assume the interest rate is the same after retirement.

a) (8 points) What are the equal withdraws he is able to take out? *Hint: You need two formulas here.*

FV

$$FV = \frac{525\left(\left(1+\frac{0.0725}{12}\right)^{12\cdot 30}-1\right)}{\left(\frac{0.0725}{12}\right)}$$

$$= 672{,}993.98$$

Amort

$$PMT = \frac{672993.98\left(\frac{0.0725}{12}\right)}{\left(1-\left(1+\frac{0.0725}{12}\right)^{-12\cdot 25}\right)} = \$4864.45$$

b) (2 points) How much did he deposit before retirement?

$$525 \cdot 12 \cdot 30 = \$189{,}000$$

c) (2 points) How much did he withdraw after retirement?

$$\$4864.45 \cdot 12 \cdot 25 = \$1{,}459{,}335$$

d) (2 points) How much interest did he earn overall?

$$\$1459335 - 189000 = \$1{,}270{,}335$$

14) (4 points each) Atrus borrowed $155,000 for a home on a 30-year loan that carried a 6.25% annual interest rate compounded monthly. After 12 years, he was able to refinance down to a 15-year loan that carried a 2.85% annual interest rate compounded monthly.

a) Determine the monthly payment for the beginning 30-year loan:

Amort

$$PMT = \frac{155000\left(\frac{0.0625}{12}\right)}{\left(1-\left(1+\frac{0.0625}{12}\right)^{-12\cdot 30}\right)} = \$954.36$$

b) How much was left on the balance after paying for 12 years? Amount Owed on a Loan

$$155000\left(1+\frac{0.0625}{12}\right)^{12\cdot 12} - \frac{954.36\left(\left(1+\frac{0.0625}{12}\right)^{12\cdot 12}-1\right)}{\left(\frac{0.0625}{12}\right)}$$

$$= \$123{,}575.37$$

c) Determine the monthly payment for the new 15-year loan:

Amort

$$PMT = \frac{123575.37\left(\frac{0.0285}{12}\right)}{\left(1-\left(1+\frac{0.0285}{12}\right)^{-12\cdot 15}\right)} = \$844.50$$

d) How much money did Atrus save by refinancing his mortgage?

$$954.36 \cdot 12 \cdot 18 - 844.50 \cdot 12 \cdot 15 = \$54{,}131.76$$

← remaining

30