1) (3 points each) Find the inverse of the following functions:

a) $\{(8,4),(0,7),(12,9),(6,6)\}$

$\{(4,8),(7,0),(9,12),(6,6)\}$

b) $f(x)=\dfrac{5}{x-3}$

① $y=\dfrac{5}{x-3}$

② $x=\dfrac{5}{y-3}$

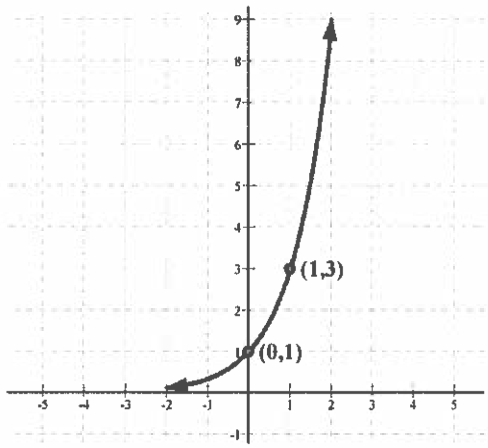
③ $xy-3x=5$

$xy=5+3x$

$y=\dfrac{5+3x}{x}$

④ $\boxed{f^{-1}(x)=\dfrac{5+3x}{x}}$

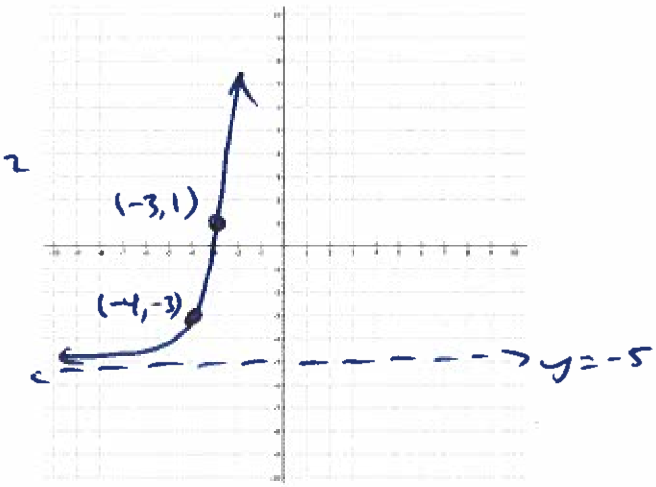
2) (5 points) Graph $g(x)=2\cdot 3^{x+4}-5$ by transforming the given function $y=3^x$. Be sure to move and label the given points and asymptotes.



left 4

multiply y-coor by 2

down 5



3) Extra credit (2 points) Find the inverse of the function $g(x)=2\cdot 3^{x+4}-5$.

① $y=2\cdot 3^{x+4}-5$

② $x=2\cdot 3^{y+4}-5$

③ $\dfrac{x+5}{2}=3^{y+4}$

$\log_3\left(\frac{x+5}{2}\right)=y+4$

$y=\log_3\left(\frac{x+5}{2}\right)-4$

④ $g^{-1}(x)=\log_3\left(\frac{x+5}{2}\right)-4$

4) (3 points) Write as one logarithm $2\log x+5\log y-4\log z$:

$=\log x^2+\log y^5-\log z^4=\log\left(\dfrac{x^2y^5}{z^4}\right)$

5) (4 points) Given that $\ln x=6$, $\ln y=10$, and $\ln z=-5$, find the exact value for $\ln\sqrt[5]{\dfrac{x^4y}{z^5}}$:

$=\frac{1}{5}\ln\left(\frac{x^4y}{z^5}\right)=\frac{1}{5}\left[\ln x^4+\ln y-\ln z^5\right]$

$=\frac{1}{5}\left[4\ln x+\ln y-5\ln z\right]$

$=\frac{1}{5}\left[4\cdot 6+10-5(-5)\right]=\boxed{\frac{59}{5}}$

$\cancel{18}$ 2 /20

6) (2 points each) Finish the explanation of the given equations. **Do not solve the equations**.

a) $5^{4x-7} = 125$ is a one-to-one exponential equation because…

because

b) $\log_{17}(2x+8) = 16$ is not a one-to-one logarithmic equation because…

becauseeeee

7) (4 points each) Solve for the variable. Be sure to find the exact value.

a) $8^{2x-1} = 16^{x+4}$

$(2^3)^{2x-1} = (2^4)^{x+4}$

$2^{6x-3} = 2^{4x+16}$

$6x-3 = 4x+16$

$2x = 19$

$x = \frac{19}{2}$

b) $6 \cdot 5^{4x+1} = 11$

$5^{4x+1} = \frac{11}{6}$

$\log_5 \frac{11}{6} = 4x+1 \Rightarrow \boxed{x = \frac{\log_5 \frac{11}{6} - 1}{4}}$

c) $\log_2(x+1) - \log_2(x+2) = \log_2 8$

$\log_2\left(\frac{x+1}{x+2}\right) = \log_2 8$

$\frac{x+1}{x+2} = 8$

$x+1 = 8x+16$

$-15 = 7x$

$x = -\frac{15}{7}$

$\emptyset$

d) $\ln(3x+4) - 3 = 12$

$\ln(3x+4) = 15$

$e^{15} = 3x+4$

$x = \frac{e^{15}-4}{3}$

8) (4 points each) After drinking six espressos in one sitting, Mike's heartrate grew exponentially without bound. Using $P(t) = P_0 e^{kt}$ where f is the heartrate in beats per minute and t is the number of minutes after he finished his 6th espresso…

Mike was able to see a hummingbird slowly flap its wings around the 4th espresso.

a) Determine the exact value for the growth rate k if his initial heart rate was 63 beats per minute but grew to 127 beats per minute after 3 minutes.

$127 = 63e^{k \cdot 3}$

$\frac{127}{63} = e^{3k} \Rightarrow 3k = \ln \frac{127}{63}$

$\Rightarrow \boxed{k = \frac{\ln 127/63}{3}}$

b) Using the exact value of k from part a, determine Mike's heartrate after 10 minutes. Round to the nearest whole number:

$P(t) = 63e^{\frac{\ln 127/63}{3} \cdot 10}$

$\approx \boxed{652 \text{ bpm}}$

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9) (2 points) Short answer: Why are logarithms necessary?

cuz

10) (2 points) What is a sequence?

cuz

11) (4 points each) Find the first four terms of the following sequences using fractions as necessary. Determine if they are arithmetic, geometric, or neither. If it is arithmetic, determine the common difference. If it is geometric, determine the common ratio.

a) $\{9n-2\}$

$n=1$ $9(1)-2=7$
$n=2$ $9(2)-2=16$
$n=3$ $9(3)-2=25$
$n=4$ $9(4)-2=34$

arithmetic $d=9$

b) $a_1=81,\ a_{n+1}=\frac{2}{3}a_n\ n\geq 1$

$a_1=81$
$a_2=\frac{2}{3}\cdot 81=54$
$a_3=\frac{2}{3}\cdot 54=36$
$a_4=\frac{2}{3}\cdot 36=24$

geometric $r=\frac{2}{3}$

12) (4 points each) Given the sequence below, find the general term a_n. For part *a* **only**, use the formula $a_n=a_1+(n-1)d$ and simplify:

a) 8, 5, 2, -1,...

$a_n=8+(n-1)(-3)$
$=-3n+3+8$
$=-3n+11$

b) $\frac{1}{\sqrt{2}},\frac{\sqrt{2}}{\sqrt{3}},\frac{\sqrt{3}}{2},\frac{2}{\sqrt{5}},\frac{\sqrt{5}}{\sqrt{6}},\ldots$

$a_n=\frac{\sqrt{n}}{\sqrt{n+1}}$

13) (3 points) Find the sum $\sum_{k=5}^{8}(-1)^k\cdot k^3$. Be sure to write out the individual terms:

$=(-1)^5\cdot 5^3+(-1)^6\cdot 6^3+(-1)^7\cdot 7^3+(-1)^8\cdot 8^3$

$=260$

14) (3 points each) Write the follow series in sigma notation. Refer to problem 12:

a) $8+5+2+(-1)+\ldots+(-28)$

$-28=-3n+11$
$-39=-3n$ $n=13$

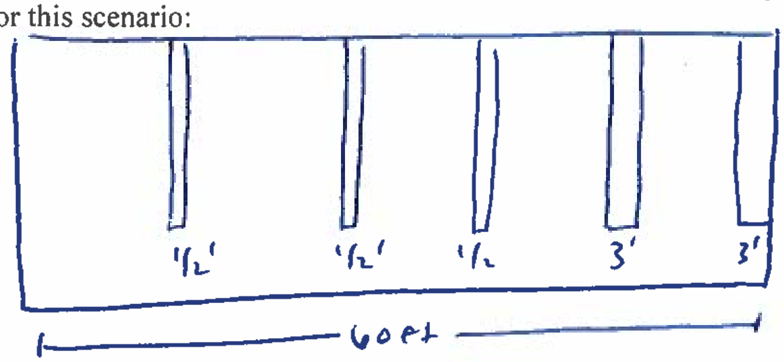
$\sum_{k=1}^{13}(-3k+11)$

b) $\frac{1}{\sqrt{2}}+\frac{\sqrt{2}}{\sqrt{3}}+\frac{\sqrt{3}}{2}+\frac{2}{\sqrt{5}}+\frac{\sqrt{5}}{\sqrt{6}}+\ldots+\frac{7}{\sqrt{50}}$

$\sum_{k=1}^{49}\frac{\sqrt{k}}{\sqrt{k+1}}$

15) (4 points each) A 60-foot long parking lot is to be painted so that it contains 5 equal-width parking spots. The right-most spot is designated for handicap only and will be flanked by a 3-foot wide line on each side to allow for the doors to open fully. The other 4 spots will be separated by a ½ foot wide line. There is no line drawn at the left edge of the parking lot.

a) Draw a picture for this scenario:



b) How wide is each of the 5 parking spots?

$$\frac{60 - (3 \cdot \frac{1}{2} + 2 \cdot 3)}{5}$$

$= \underline{10.5 \text{ ft}}$

c) Starting your count from the left-most edge of the parking lot, determine the location of the left-edge of the ½ foot wide lines:

$10.5', 21.5', 32.5'$

+11 ft   +11 ft

16) (4 points each) Evaluate the sums using either formula: $S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$ or $S_n = \frac{n}{2}\left(a_1 + a_n\right)$.

Write answers as improper fractions as needed:

a) $\sum_{k=1}^{500}(5k+10)$

$a_1 = 5(1)+10 = 15$

$a_{500} = 5(500)+(10) = 2510$

$n = 500$

$S_{500} = \frac{500}{2}(15+2510) = \boxed{631250}$

b) $\sum_{k=42}^{58}\left(\frac{2k-1}{6}\right)$

$a_1 = \frac{2(42)-1}{6} = \frac{83}{6}$

$a_{17} = \frac{2(58)-1}{6} = \frac{115}{6}$

$n = 58-42+1 = 17$

$S_{17} = \frac{17}{2}\left(\frac{83}{6} + \frac{115}{6}\right)$

$= \boxed{\frac{561}{2} \text{ or } 280.5}$

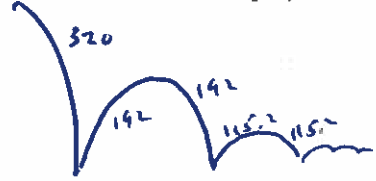
17) (5 points) Leo Wong's daughter tosses the pony she's always wanted as a child off of a 320 foot cliff. Thankfully, the pony is made of rubber and bounces gleefully, rebounding $\frac{3}{5}$ of the distance fallen. How far vertically does the pony travel? Use the formula $S_\infty = \frac{a_1}{1-r}$ in your answer.

drop $\frac{320}{1-3/5}$

$+$

rebound $\frac{192}{1-3/5}$

$= \underline{1280 \text{ ft}}$



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