1) (2 points) What are the four ways to solve a quadratic equation?

✓

2) (4 points each) Solve for the variable:

a) $x^2 - 36 = 0$

$x^2 = 36$

$x = \pm 6$

b) $-4(x+1)^2 + 6 = 5$

$-4(x+1)^2 = -1$

$(x+1)^2 = \frac{1}{4}$

$x+1 = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$x = -1 \pm \frac{1}{2} \rightarrow$ $x = -1/2$, $x = -3/2$

3) (3 points each) Fill the blank with the number necessary to complete the square and then factor. Show all necessary work:

a) $x^2 - 20x + \underline{100}$

$\left(\frac{-20}{2}\right)^2$

$(x-10)^2$

b) $x^2 + x + \underline{\frac{1}{4}}$

$\left(\frac{1}{2}\right)^2$

$\left(x+\frac{1}{2}\right)^2$

4) (5 points each) Solve by completing the square:

a) $x^2 + 8x + 12 = 0$

$x^2 + 8x + \underline{16} = -12 + \underline{16}$

$(x+4)^2 = 4$

$x+4 = \pm 2$

$x = -4 \pm 2 \rightarrow$ -2, -6

b) $3x^2 + 8x + 12 = 0$

$x^2 + \frac{8}{3}x + \underline{\frac{16}{9}} = -4 + \underline{\frac{16}{9}}$

$\left(x+\frac{4}{3}\right)^2 = -\frac{20}{9}$ $\quad x+\frac{4}{3} = \pm\sqrt{-\frac{20}{9}} = \pm\frac{2\sqrt{5}}{3}i$

$x = -\frac{4}{3} \pm \frac{2\sqrt{5}}{3}i$

5) (5 points each) Solve by using the quadratic formula:

a) $x^2 - 4x + 9 = 0$

$a=1 \quad b=-4 \quad c=9$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(9)}}{2(1)}$

$x = \frac{4 \pm \sqrt{-20}}{2} = \frac{4 \pm 2\sqrt{5}i}{2} = 2 \pm \sqrt{5}i$

b) $3x^2 + 8x + 12 = 0$

$a=3 \quad b=8 \quad c=12$

$x = \frac{-8 \pm \sqrt{8^2 - 4(3)(12)}}{2(3)} = \frac{-8 \pm \sqrt{-80}}{6}$

$x = \frac{-8 \pm 4\sqrt{5}i}{6} = -\frac{4}{3} \pm \frac{2\sqrt{5}}{3}i$

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6) (6 points) The wildly unpopular musical "Catcher in the Wheat" is causing theater managers to increase ticket prices to make ends meet. Currently, the prices are set at $14.00 per ticket and brings in an average crowd of 560 per show. They've found that, for every $2.00 the ticket prices increase, they will lose 10 people. If they need to bring in $19,000 per show to stay afloat, what should the new ticket prices be? Let x be the number of times the prices are increased. *Note: There are two possible answers. Do they both work for this situation?*

x = # of $2.00 ticket price increases

$(14+2x)(560-10x) = 19000$

$-20x^2 + 980x + 7840 = 19000$

$0 = 20x^2 - 980x + 11{,}160$

$0 = x^2 - 49x + 558$

$0 = (x-18)(x-31)$

$x = 18 \quad x = 31$

both work!

$50 or $76

7) (5 points each) Solve for the variable:

a) $x^4 - 13x^2 + 12 = 0$

$a = x^2 \quad a^2 - 13a + 12 = 0$

$(a-12)(a-1) = 0$

$a = 12 \quad a = 1 \rightarrow x^2 = 1$

$x^2 = 12 \Rightarrow x = \pm 2\sqrt{3} \quad x = \pm 1$

b) $4x^{\frac{2}{5}} + 5x^{\frac{1}{5}} - 6 = 0$

$b = x^{1/5} \quad 4b^2 + 5b - 6 = 0$

$(4b-3)(b+2) = 0$

$b = 3/4 \quad b = -2$

$x^{\frac{1}{5}} = 3/4 \Rightarrow x = \frac{243}{1024} \quad x^{\frac{1}{5}} = -2 \Rightarrow x = -32$

8) (2 points) Where is the vertex of the quadratic function $f(x) = -3(x+6)^2 + 7$ located?

$(-6, 7)$

9) (3 points) Explain the transformations necessary to sketch the graph of $f(x) = -3(x-7)^2 + 1$:

1) Right 7
2) Vertical stretch by 3
3) Vertical reflection over x-axis
4) up 1

10) (3 points each) For the function $f(x) = x^2 + 2x - 8$, find...

a) The vertex:

$x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$f(-1) = (-1)^2 + 2(-1) - 8 = -9$

$(-1, -9)$

b) The x-intercepts:

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4 \quad (-4, 0)$

$x = 2 \quad (2, 0)$

c) The y-intercept:

$f(0) = 0^2 + 2 \cdot 0 - 8 = -8$

$(0, -8)$

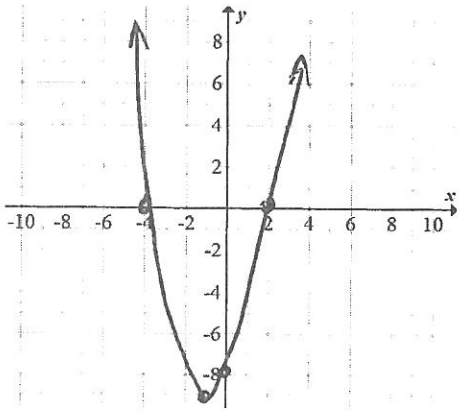
d) The domain:

$\mathbb{R}$

e) The range:

$[-9, \infty)$

f) The graph:



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11) (3 points each) After retiring for the tough javelin-throwing lifestyle, Jo Jo Ba decides to take up leftover carved pumpkin throwing. A particular pumpkin follows a path given by the equation $h(t) = -16t^2 + 48t + 10$ where t is time in seconds and h is height off of the ground.

a) Find the time at which the pumpkin will the highest off the ground:

$t = -\frac{b}{2a} = \frac{-48}{2(-16)} = 1.5$ sec

b) Find the highest height the pumpkin will be in the air.

$h(1.5) = -16(1.5)^2 + 48(1.5) + 10$

$= 46$ feet

c) Find the time when the pumpkin will hit the ground. Round to two places:

$-16t^2 + 48t + 10 = 0$

$8t^2 - 24t - 5 = 0$

$t = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(8)(-5)}}{2(8)}$

$t \approx 3.20$ sec

~~$t \approx -0.20$~~

12) (3 points each) For the function $f(x) = 4 \cdot 2^{x-5} - 6$...

a) Describe the transformations necessary to sketch the graph

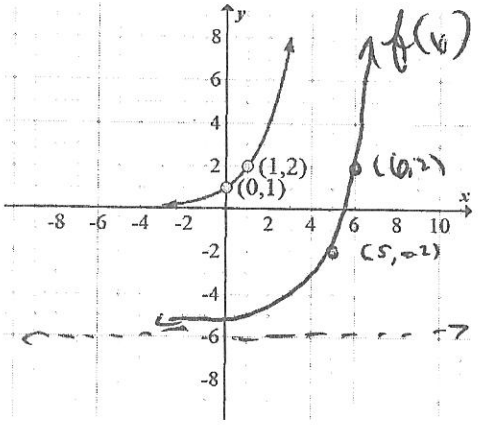
1) Right 5
2) vertical stretch by 4
3) down 6

$(0,1) \to (5,1) \to (5,4) \to (5,-2)$

$(1,2) \to (6,2) \to (6,8) \to (6,2)$

$y = 0 \to y = -6$

b) Sketch a graph of the transformation using the graph $y = 2^x$ below. Be sure to label the transformed points and asymptote:



13) (5 points each) Solve for the variable in the equations:

a) $2^{4x-8} = 16$

$2^{4x-8} = 2^4$

$4x - 8 = 4$

$x = 3$

b) $5 \cdot 125^{3x-1} = 25^{2x+1}$

$5 \cdot (5^3)^{3x-1} = (5^2)^{2x+1}$

$5 \cdot (5)^{9x-3} = 5^{4x+2}$

$5^{9x-2} = 5^{4x+2}$

$9x - 2 = 4x + 2$

$5x = 4$

$x = 4/5$

4/25