

1) (2 points) What are the four ways to solve a quadratic equation?

- 1) Steal socks
- 2) Dance
- 3) ?
- 4) Profit

2) (4 points each) Solve for the variable:

a)  $x^2 - 25 = 0$

$$x^2 = 25$$

$$x = \pm 5$$

b)  $4(x-2)^2 + 6 = 10$

$$4(x-2)^2 = 4$$

$$(x-2)^2 = 1$$

$$x-2 = \pm 1$$

$$x = 2 \pm 1 \rightarrow \begin{cases} 2+1 = 3 \\ 2-1 = 1 \end{cases}$$

3) (3 points each) Fill the blank with the number necessary to complete the square and then factor. Show all necessary work:

a)  $x^2 + 8x + \underline{\hspace{2cm}}$

$$\left(\frac{8}{2}\right)^2 = 4^2 = 16$$

$$(x+4)^2$$

b)  $x^2 - 17x + \underline{\hspace{2cm}}$

$$\left(\frac{-17}{2}\right)^2 = \frac{289}{4}$$

$$\left(x - \frac{17}{2}\right)^2$$

4) (5 points each) Solve by completing the square:

a)  $x^2 + 6x + 4 = 0$

$$x^2 + 6x + 9 = -4 + 9$$

$$(x+3)^2 = 5$$

$$x+3 = \pm\sqrt{5}$$

$$x = -3 \pm \sqrt{5}$$

b)  $4x^2 - 5x - 6 = 0$

$$4x^2 - 5x = 6$$

$$x^2 - \frac{5}{4}x + \frac{25}{64} = \frac{3}{2} + \frac{25}{64}$$

$$\left(x - \frac{5}{8}\right)^2 = \frac{121}{64}$$

$$x - \frac{5}{8} = \pm \frac{11}{8}$$

$$x = \frac{5}{8} \pm \frac{11}{8} \rightarrow \begin{cases} \frac{5}{8} + \frac{11}{8} = 2 \\ \frac{5}{8} - \frac{11}{8} = -\frac{3}{4} \end{cases}$$

5) (5 points each) Solve by using the quadratic formula:

a)  $x^2 + 6x + 4 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{20}}{2} = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$= -3 \pm \sqrt{5}$$

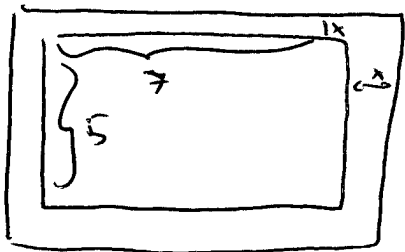
b)  $4x^2 - 5x - 6 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-6)}}{2(4)}$$

$$= \frac{5 \pm \sqrt{121}}{8} = \frac{5 \pm 11}{8} \rightarrow \begin{cases} \frac{5+11}{8} = 2 \\ \frac{5-11}{8} = -\frac{6}{8} = -\frac{3}{4} \end{cases}$$

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- 6) (5 points) A 5x7 photo will have a matte that has a uniform width. If the entire area of the photo with the matte is to be 55.25 square inches, how wide should the matte be?



$$(7 + 2x)(5 + 2x) = 55.25$$

$$4x^2 + 24x + 35 = 55.25$$

$$4x^2 + 24x - 20.25 = 0$$

$$x = \frac{-24 \pm \sqrt{24^2 - 4(4)(-20.25)}}{2(4)}$$

$$= \frac{-24 \pm \sqrt{900}}{8} = \frac{-24 \pm 30}{8}$$

this  $\rightarrow$   $\left(\frac{3}{4}\right)$   $\left(-\frac{27}{4}\right)$

- 7) (5 points each) Solve for the variable:

a)  $x^4 - 9x^2 + 8 = 0$

$$(x^2)^2 - 9(x^2) + 8 = 0$$

let  $u = x^2$   $u^2 - 9u + 8 = 0$

$$(u-8)(u-1) = 0$$

$$u = 8 \quad u = 1$$

$$x^2 = 8 \quad x^2 = 1$$

$$x = \pm 2\sqrt{2}$$

$$x = \pm 1$$

b)  $2x - 7\sqrt{x} + 3 = 0$

$$2(\sqrt{x})^2 - 7(\sqrt{x}) + 3 = 0$$

let  $m = \sqrt{x}$

$$2m^2 - 7m + 3 = 0$$

$$(2m-1)(m-3) = 0$$

$$m = \frac{1}{2} \quad m = 3$$

$$\sqrt{x} = \frac{1}{2} \Rightarrow x = \frac{1}{4}$$

$$\sqrt{x} = 3 \Rightarrow x = 9$$

- 8) (3 points) Where is the vertex of the quadratic function  $f(x) = -\frac{14}{7}(x-6)^2 - 8$  located?

$$(6, -8)$$

- 9) (3 points) Explain the transformations necessary to sketch the graph of  $f(x) = -2(x+3)^2 + 4$ :

1)  $h = -3$  left 3

2)  $k = 4$  up 4

2)  $a = -2$  multiply y-coord by -2 or reflect vertically & stretch

- 10) (3 points each) For the function  $f(x) = x^2 + 2x - 3$ , find...

- a) The vertex:

$$x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$$

$$f(-1) = (-1)^2 + 2(-1) - 3 = -4$$

- c) The y-intercept:

$$f(0) = 0^2 + 2 \cdot 0 - 3 = -3$$

$$(0, -3)$$

- e) The range:

$$[-4, \infty)$$

- b) The x-intercepts:

$$x^2 + 2x - 3 = 0$$

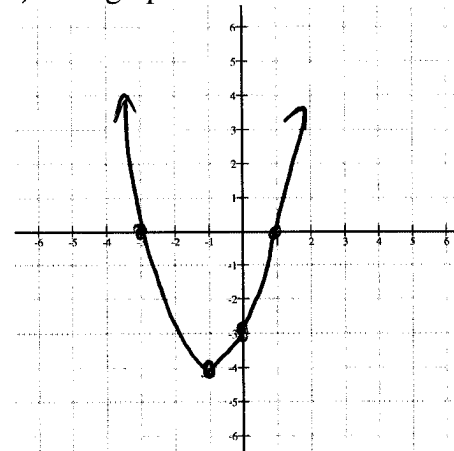
$$(x+3)(x-1) = 0$$

$$x = -3, 1$$

- d) The domain:

$$\mathbb{R}$$

- f) The graph:



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11) (3 points each) A large cupcake thrown into the air off of a 47-foot cliff follows a path given by the function  $h(t) = -16t^2 + 32t + 47$  where  $h$  is the height in feet and  $t$  is time in seconds:

a) Find the time at which the cupcake will be the highest off the ground:

$$t = -\frac{b}{2a} = -\frac{32}{2(-16)} = 1 \text{ second}$$

b) Find the highest height the cupcake will be in the air.

$$h(1) = -16(1)^2 + 32(1) + 47 = 63 \text{ feet}$$

c) Find the time when the cupcake will hit the ground. Round your answer to two decimal places:

$$-16t^2 + 32t + 47 = 0$$

$$t = \frac{-32 \pm \sqrt{32^2 - 4(-16)(47)}}{2(-16)} \approx 2.98 \text{ sec}$$

~~-0.90~~



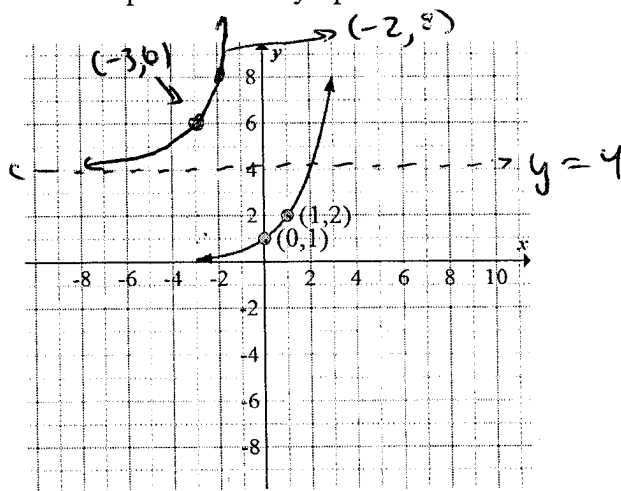
University research has shown that children actually hate small cupcakes.

12) (3 points each) For the function  $f(x) = 2 \cdot 2^{x+3} + 4 \dots$

a) Describe the transformations necessary to sketch the graph

- 1)  $h = -3$  left 3
- 2)  $a = 2$  vertically stretch by 2 or multiply y-coordinate
- 3)  $k = 4$  up 4

b) Sketch a graph of the transformation using the graph  $y = 2^x$  below. Be sure to label the transformed points and asymptote:



13) (5 points each) Solve for the variable in the equations:

a)  $3^{2x-7} = 81$

$$3^{2x-7} = 3^4$$

$$2x - 7 = 4$$

$$x = \frac{11}{2}$$

b)  $16^{5x-9} = 32^{12x+6}$

$$(2^4)^{5x-9} = (2^5)^{12x+6}$$

$$2^{20x-36} = 2^{60x+30}$$

$$20x - 36 = 60x + 30$$

$$-66 = 40x$$

$$x = \frac{-66}{40} = \left[ \frac{-33}{20} \right]$$

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