

1) (3 points) How do you find the domain of a rational function?

thinking emoji

2) (4 points) Find the domain of the function  $f(x) = \frac{x^2 + 4x + 4}{x^2 + 9x + 8}$ :

$$x^2 + 9x + 8 = 0$$

$$(x+8)(x+1) = 0$$

$$x \neq -8, -1$$

3) (5 points each) Perform the indicated operation. You may leave the answer in factored form:

a)  $\frac{x^2 - 2x - 3}{x^2 - 6x - 7} \cdot \frac{x^2 - 5x - 14}{4x^2 + 8x}$

$$\frac{(x-3)\cancel{(x+1)}}{\cancel{(x-7)}(x+1)} \cdot \frac{\cancel{(x-7)}\cancel{(x+2)}}{4x(x+2)}$$

$$\boxed{\frac{x-3}{4x}}$$

b)  $\frac{10x - 50}{x^2 + 6x + 9} \div \frac{x^2 - 25}{x^2 + 8x + 15}$

$$\frac{10\cancel{(x-5)}}{(x+3)^2} \cdot \frac{\cancel{(x+3)}\cancel{(x+5)}}{\cancel{(x+5)}(x+3)} = \boxed{\frac{10}{x+3}}$$

4) (5 points each) Perform the indicated operation. Be sure to simplify the numerator and reduce as needed. You may leave the denominator factored:

a)  $\frac{x}{x^2 + 4x + 4} - \frac{x+1}{x+2} \cdot \frac{(x+2)}{(x+2)}$

LCD  $\rightarrow (x+2)^2$

$$\frac{x - (x+1)(x+2)}{(x+2)^2} = \frac{x - x^2 - 3x - 2}{(x+2)^2}$$

$$\boxed{\frac{-x^2 - 2x - 2}{(x+2)^2}}$$

b)  $\frac{(x+5)x+1}{(x+3)^2} + \frac{x+2}{x^2+8x+15} \cdot \frac{(x+3)}{(x+3)}$

LCD:  $(x+3)^2(x+5)$

$$\frac{(x+5)(x+1) + (x+2)(x+3)}{(x+3)^2(x+5)} = \frac{x^2 + 6x + 5 + x^2 + 5x + 6}{(x+3)^2(x+5)}$$

$$= \boxed{\frac{2x^2 + 11x + 11}{(x+3)^2(x+5)}}$$

5) (5 points each) Simplify the complex fractions completely.

LCD:  $(x+1)(x-1)$

a)  $\frac{(x+1)(x-1)}{(x+1)(x-1)} \cdot \frac{1}{x+1} + \frac{2}{(x+1)(x-1)} \cdot \frac{(x-1)}{(x-1)}$

$\frac{(x+1)(x-1)}{(x+1)(x-1)} \cdot \frac{2}{x+1} + \frac{1}{x-1} \cdot \frac{(x+1)(x-1)}{(x+1)(x-1)}$

$$= \frac{x-1 + 2(x+1)}{2(x-1) + x+1}$$

$$= \frac{x-1+2x+2}{2x-2+x+1} = \boxed{\frac{3x+1}{3x-1}}$$

LCD:  $(x+3)(x-3)$

b)  $\frac{(x+3)(x-3)}{(x+3)(x-3)} \cdot \frac{x+1}{x^2-9} + 5 \cdot \frac{(x+3)(x-3)}{(x+3)(x-3)}$

$\frac{(x+3)(x-3)}{(x+3)(x-3)} \cdot \frac{1}{x+3} + \frac{5(x^2-9)}{(x+3)(x-3)}$

$$= \frac{x+1+5(x^2-9)}{x-3}$$

$$= \boxed{\frac{5x^2+x-44}{x-3}}$$

LCD:  $a^2$

$$c) \frac{a^{-2}+b}{7-a^{-1}} = \frac{a^2 \cdot \frac{1}{a^2} + b \cdot a^2}{a^2 \cdot 7 - \frac{1}{a} \cdot a^2}$$

$$= \boxed{\frac{1+a^2b}{7a^2-a}}$$

4/2

6) (5 points each) Solve for the variable:  
 LCD  
 $(x+4)(x-7)$   
 a)  $\frac{2}{x+4} + \frac{x(x+4)(x-7) \cdot 2}{x-7} = \frac{2}{x^2-3x-28}$

$$2(x-7) + x(x+4) = 2$$

$$x^2 + 6x - 16 = 0$$

$$(x+8)(x-2) = 0$$

$$\boxed{x = -8, 2}$$

LCD  
 $(x+3)(x-6)$   $(x+3)(x-6)$   
 b)  $\frac{5}{x+3} + \frac{8}{x-6} = \frac{x^2+6x}{x^2-3x-18}$

$$5(x-6) + 8(x+3) = x^2+6x$$

$$0 = x^2 - 7x + 6$$

$$0 = (x-6)(x-1)$$

$$\cancel{x=6} \quad \boxed{x=1}$$

7) (2 points each) Short Answer: In this chapter, several of the sections dealt with using the LCD to complete the problem. Explain in each type of problem how the LCD was used. Do not give examples. Instead, give instructions as if you were explaining the process to someone who did not know:

a) Adding/Subtracting Fractions:

making pancakes

b) Simplifying Complex Fractions:

making bacon pancakes

c) Solving Equations with Rational Expressions:

take some bacon and I'll put it in a pancake

8) (5 points) Solve for  $k$  in the equation  $\frac{2}{k} + \frac{3}{l} = \frac{1}{m+1}$ :  
 LCD  $k l (m+1)$

$$2l(m+1) + 3k(m+1) = kl$$

$$2l(m+1) = kl - 3k(m+1)$$

$$2l(m+1) = k[l - 3(m+1)]$$

factor out  $k$

divide

$$\boxed{\frac{2l(m+1)}{l-3(m+1)} = k}$$

9) (3 points each) Consider the two problems below. Give the steps necessary to complete the problem. Do not actually complete the problem. Hint: Think about the directions that would be given for each problem type.

a)  $\frac{x^2+5x+6}{4x+4} - \frac{x+1}{x^2+2x+1}$

make one fraction by rewriting both fractions to have the LCD as the denominator and simplify the numerators.

b)  $\frac{x^2+5x+6}{4x+4} - \frac{x+1}{x^2+2x+1} = 5$

multiply both sides by the LCD to clear out fractions. Solve for the variable

	$d$	$r$	$t$
S B	$d - 80$	$r$	$2 \Rightarrow d - 80 = 2r \Rightarrow d = 2r + 80$
PAT	$d$	$2r$	$2 \Rightarrow d = 4r$

- 10) (5 points each) SpongeBob and Patrick decide to race boats starting from the same place (despite the fact that SpongeBob does not have his boating license yet). 2 hours into the race, SpongeBob is 80 miles behind Patrick. Patrick is going twice as fast as SpongeBob.
- a) How fast are they each going?      b) How far did each get after 2 hours?

$$2r + 80 = 4r$$

$$80 = 2r$$

$r = 40 \text{ mph}$ S B	$2r = 80$ PAT
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PAT  $d = 4r = 4(40) = 160 \text{ mi PAT}$

S B  $d - 80 = 80 \text{ mi S B}$

- 11) (6 points) You and your siblings hold the most-awesome Math party of the year at your parents' home and end up making a mess in the process. Your parents, who you thought were away on vacation, are coming home in 5 hours. From past experience, you know that you can clean the house in 9 hours, your brother can do the job in 10 hours and your sister can do the job in 8 hours. If you work together, will you be able to clean the house before your parents get home? Be sure to show all necessary work to solve the problem.

	No. hours to complete	Part done in 1 hr
You	9	$\frac{1}{9}$
Bro	10	$\frac{1}{10}$
Sis	8	$\frac{1}{8}$
Tag	$x$	$\frac{1}{x}$

$$\frac{1}{9} + \frac{1}{10} + \frac{1}{8} = \frac{1}{x} \quad \text{LCM} = 360x$$

$$40x + 36x + 45x = 360$$

$$121x = 360$$

$$x \approx 2.975$$

- 12) (5 points) Suppose that  $a$  varies directly at the square of  $b$ . When  $a$  is 2,  $b$  is 5. Find the value of  $a$  when  $b$  is 7:

$$a = k \cdot b^2$$

$$2 = k \cdot 5^2 \Rightarrow k = \frac{2}{25}$$

$$a = \frac{2}{25} b^2$$

$$a = \frac{2}{25} \cdot 7^2 = \frac{98}{25}$$

- 13) (5 points each) Goo-gon, a mystic warrior, swings a spiked ball on a chain above his head before he delivers a crushing blow to his enemy Mr. Sticky. See the illustration below. The tension  $T$  in the chain is directly proportional to the square of the speed  $s$  of the ball and inversely proportional to the radius  $r$  of the circle. If the tension in the chain is 24 pounds when the speed of the ball is 6 feet per second and the radius is 3 feet, find...

- a) An equation for this situation including a value for  $k$ :      b) The tension when the speed is 8 feet per second and the radius is 2.5 feet:

$$T = \frac{k s^2}{r}$$

$$24 = \frac{k \cdot 6^2}{3} \Rightarrow k = 2$$

$$T = \frac{2 \cdot 8^2}{2.5} = 51.2 \text{ lbs}$$

$$T = \frac{2s^2}{r}$$