

1) (4 points each) Simplify:

a)  $\tan x (\tan x + \cot x)$

$$\begin{aligned} &= \tan^2 x + \underbrace{\tan x \cdot \cot x}_{\text{cancel}} \\ &= \tan^2 x + 1 \\ &= \boxed{\sec^2 x} \end{aligned}$$

b)  $\frac{\tan x}{\tan x + \cot x}$

$$\begin{aligned} &= \frac{\sin x}{\cos x} - \sin x \cos x \\ &\quad \left( \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) \cdot \sin x \cos x \\ &= \frac{\sin^2 x}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x}{1} = \boxed{\sin^2 x} \end{aligned}$$

2) (4 points each) Find the exact value of  $\cos \frac{\pi}{12}$  using the given methods.

a) A Sum or Difference Formula:

$$\begin{aligned} &\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}} \end{aligned}$$

b) A Half Angle Formula:

$$\begin{aligned} \cos \frac{\pi}{12} &= \cos \frac{\frac{\pi}{6}}{2} \\ &= \sqrt{\frac{1 + \cos \frac{\pi}{6}}{2}} = \sqrt{\frac{1 + \sqrt{3}/2}{2}} \\ &= \boxed{\frac{\sqrt{2 + \sqrt{3}}}{2}} \end{aligned}$$

c) Using your answer from either part a or b above, explain how you can find the exact value of  $\cos\left(\frac{13\pi}{12}\right)$  by using  $\frac{\pi}{12}$  as a reference angle:

Hmm..

3) (5 points each) Simplify:

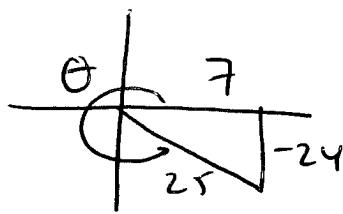
a)  $\frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{\cos(\alpha + \beta) - \cos(\alpha - \beta)}$

$$\begin{aligned} &= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta - (\sin \alpha \cos \beta - \cos \alpha \sin \beta)}{\cos \alpha \cos \beta - \sin \alpha \sin \beta - (\cos \alpha \cos \beta + \sin \alpha \sin \beta)} \\ &= \frac{2 \cos \alpha \sin \beta}{-2 \sin \alpha \sin \beta} = \boxed{-\cot \alpha} \end{aligned}$$

b)  $\frac{\sin^2 \alpha}{\tan^2 \alpha} - \frac{\tan^2 \alpha}{\sec^2 \alpha} = \frac{\sin^2 \alpha}{\frac{\sin^2 \alpha}{\cos^2 \alpha}} - \frac{1}{\frac{\cos^2 \alpha}{\cos^2 \alpha}}$

$$\begin{aligned} &= \cos^2 \alpha - \sin^2 \alpha \\ &= \boxed{\cos 2\alpha} \end{aligned}$$

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4) (5 points each) Given  $\cos\theta = \frac{7}{25}$ , where  $\theta$  is in Quadrant IV, find the exact values for...

a)  $\sin(2\theta)$

$$\begin{aligned} &= 2\sin\theta\cos\theta \\ &= 2\left(-\frac{24}{25}\right)\left(\frac{7}{25}\right) = \boxed{-\frac{336}{625}} \end{aligned}$$

b)  $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$$\begin{aligned} &= \left(\frac{7}{25}\right)^2 - \left(-\frac{24}{25}\right)^2 \\ &= \boxed{-\frac{527}{625}} \end{aligned}$$

c)  $\tan(2\theta)$

$$\begin{aligned} &= \frac{\sin 2\theta}{\cos 2\theta} = \frac{-336/625}{-527/625} \\ &= \boxed{\frac{336}{527}} \end{aligned}$$

d) The Quadrant where  $2\theta$  resides. Explain why:

Q3

$\sin 2\theta < 0$

$\cos 2\theta < 0$

$\tan 2\theta > 0$

5) (6 points part a, 9 points part b) Prove the following identities:

a)  $\tan^2\theta\sin^2\theta = \tan^2\theta + \cos^2\theta - 1$

b)  $4\cos^2x - 4 + \sec^2x = \cos^2x - 2\sin^2x + \sin^2x\tan^2x$

$$\begin{aligned} &\tan^2\theta(1-\cos^2\theta) \\ &\tan^2\theta - (\tan^2\theta\cos^2\theta) \\ &\tan^2\theta - \sin^2\theta \\ &\tan^2\theta - (1-\cos^2\theta) \\ &\tan^2\theta - 1 + \cos^2\theta \end{aligned}$$

$$\begin{aligned} &4\cos^2x - 4 + \frac{1}{\cos^2x} \\ &\frac{4\cos^4x - 4\cos^2x + 1}{\cos^2x} \\ &\text{double angle} \\ &\frac{(2\cos^2x - 1)^2}{\cos^2x} \\ &\frac{(\cos^2x - \sin^2x)^2}{\cos^2x} \end{aligned}$$

$$\frac{\cos^4x - 2\cos^2x\sin^2x + \sin^4x}{\cos^2x}$$

$$= \cos^2x - 2\sin^2x + \sin^2x\tan^2x$$

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6) (4 points each) Find the exact values or explain why it does not exist:

a)  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

b)  $\cos(\cos^{-1}(-1.1))$

DNE. -1.1 isn't in the domain of cosine inverse

c)  $\cos^{-1}\left(\cos\left(-\frac{2\pi}{3}\right)\right)$

$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$

d)  $\sin\left(\sin^{-1}\left(-\frac{4}{5}\right) - \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= \sin(\sin^{-1}(-\frac{4}{5}))\cos(\cos^{-1}(-\frac{\sqrt{3}}{2})) - \cos(\sin^{-1}(-\frac{4}{5}))\sin(\cos^{-1}(-\frac{\sqrt{3}}{2}))$

$= \left(-\frac{4}{5}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{3}{5}\right)\left(\frac{1}{2}\right) = \frac{4\sqrt{3}-\sqrt{3}}{10}$

7) (6 points each) Solve for the variable:

a)  $\cos^2 x - 1 = 0$  (Hint: You need a  $k$ )

$\cos^2 x = 1 \Rightarrow \cos x = \pm 1$

$x = 0 + k\cdot\pi, k \in \mathbb{Z}$

$2x = \frac{\pi}{3} + 2\pi k \Rightarrow x = \frac{\pi}{6} + \pi k$

$2x = \frac{2\pi}{3} + 2\pi k \Rightarrow x = \frac{\pi}{3} + \pi k$

$x \in \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3} \right\}$

8) Fill in the blank using interval notation:

Sin x	Cos x	Tan x	Sin <sup>-1</sup> x	Cos <sup>-1</sup> x	Tan <sup>-1</sup> x
Domain	this answer left blank for you				
Range					

\*Write the domain restrictions for these three functions.

9) (2 points) Explain why we restricted the domains of  $y = \sin x$ ,  $y = \cos x$ , and  $y = \tan x$  in this chapter.

perché no?

Extra Credit:

Simplify:  $-\sin\left(\frac{\pi}{3} - \alpha\right)\sin\left(\frac{\pi}{3} + \alpha\right) + \cos\left(\frac{\pi}{3} - \alpha\right)\cos\left(\frac{\pi}{3} + \alpha\right)$

$- \frac{1}{2}$