MATH 1540 Summer 2019 **Mike McCraith**
Exam 2

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

- **Write your name below on the space provided.**
- **This test has a total of 6 pages. Feel free to tear off the last page. I do not need it returned.**
- **Work the problem in the space provided. If you need more space, write on the back of the test. Be sure to mark on the test that work is continued on the back. Be sure to number your work.**
- **To insure maximum credit, show your work. In general, full credit will not be given for unsupported answers.**
- **Be sure to write neatly. All tests must be done in pencil. If I cannot read what was written or it was no done in pencil, do not expect the problem to be graded.**
- **If you finish early, go over the test again.**

Good luck!

Number	Maximum	Score
1	8	
2	12	
3	10	
4	20	
5	15	
6	16	
7	12	
8	5	
9	2	
Total	100	

Name _____

CIRCLE FINAL ANSWERS

1) (4 points each) Simplify:

a) $\tan x\left(\tan x+\cot x\right)$

b) $\dfrac{\tan x}{\tan x+\cot x}$

2) (4 points each) Find the exact value of $\cos\dfrac{\pi}{12}$ using the given methods.

a) A Sum or Difference Formula:

b) A Half Angle Formula:

c) Using your answer from either part *a* or *b* above, **explain** how you can find the exact value of $\cos\left(\dfrac{13\pi}{12}\right)$ by using $\dfrac{\pi}{12}$ as a reference angle.

3) (5 points each) Simplify:

a) $\dfrac{\sin\left(\alpha+\beta\right)-\sin\left(\alpha-\beta\right)}{\cos\left(\alpha+\beta\right)-\cos\left(\alpha-\beta\right)}$

b) $\dfrac{\sin^2\alpha}{\tan^2\alpha}-\dfrac{\tan^2\alpha}{\sec^2\alpha}$

4) (5 points each) Given $\cos\theta = \frac{7}{25}$, where θ is in Quadrant IV, find the exact values for…

a) $\sin(2\theta)$

b) $\cos(2\theta)$

c) $\tan(2\theta)$

d) The Quadrant where 2θ resides. Explain why:

5) (6 points part *a*, 9 points part *b*) Prove the following identities:

a) $\tan^2\theta\sin^2\theta = \tan^2\theta + \cos^2\theta - 1$

b) $4\cos^2 x - 4 + \sec^2 x = \cos^2 x - 2\sin^2 x + \sin^2 x\tan^2 x$

6) (4 points each) Find the exact values or explain why it does not exist:

a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

b) $\cos\left(\cos^{-1}(-1.1)\right)$

c) $\cos^{-1}\left(\cos\left(-\frac{2\pi}{3}\right)\right)$

d) $\sin\left(\sin^{-1}\left(-\frac{4}{5}\right)-\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

7) (6 points each) Solve for the variable:

a) $\cos^2 x-1=0$ (Hint: You need a k)

b) $\sin(2x)=\frac{\sqrt{3}}{2}$ on $[0,2\pi)$

8) Fill in the blank using interval notation:

	$\sin x$ *	$\cos x$ *	$\tan x$ *	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain						
Range						

*Write the domain restrictions for these three functions.

9) (2 points) Explain why we restricted the domains of $y=\sin x$, $y=\cos x$, and $y=\tan x$ in this chapter.

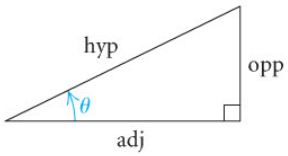
Extra Credit:

Simplify: $-\sin\left(\frac{\pi}{3}-\alpha\right)\sin\left(\frac{\pi}{3}+\alpha\right)+\cos\left(\frac{\pi}{3}-\alpha\right)\cos\left(\frac{\pi}{3}+\alpha\right)$

Trigonometry

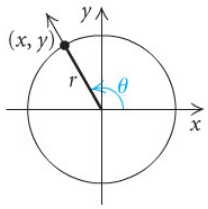
Trigonometric Functions

Acute Angles



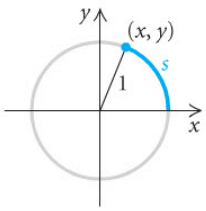
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \csc\theta = \frac{\text{hyp}}{\text{opp}},$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \sec\theta = \frac{\text{hyp}}{\text{adj}},$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}}, \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Any Angle



$$\sin\theta = \frac{y}{r}, \quad \csc\theta = \frac{r}{y},$$
$$\cos\theta = \frac{x}{r}, \quad \sec\theta = \frac{r}{x},$$
$$\tan\theta = \frac{y}{x}, \quad \cot\theta = \frac{x}{y}$$

Real Numbers



$$\sin s = y, \quad \csc s = \frac{1}{y},$$
$$\cos s = x, \quad \sec s = \frac{1}{x},$$
$$\tan s = \frac{y}{x}, \quad \cot s = \frac{x}{y}$$

Basic Trigonometric Identities

$$\sin(-x) = -\sin x,$$
$$\cos(-x) = \cos x,$$
$$\tan(-x) = -\tan x,$$

$$\tan x = \frac{\sin x}{\cos x},$$
$$\cot x = \frac{\cos x}{\sin x},$$

$$\csc x = \frac{1}{\sin x},$$
$$\sec x = \frac{1}{\cos x},$$
$$\cot x = \frac{1}{\tan x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1,$$
$$1 + \cot^2 x = \csc^2 x,$$
$$1 + \tan^2 x = \sec^2 x$$

Identities Involving $\pi/2$

$$\sin(\pi/2 - x) = \cos x,$$
$$\cos(\pi/2 - x) = \sin x, \quad \sin(x \pm \pi/2) = \pm\cos x,$$
$$\tan(\pi/2 - x) = \cot x, \quad \cos(x \pm \pi/2) = \mp\sin x$$

Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v,$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v,$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x,$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1,$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}, \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}},$$
$$\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

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