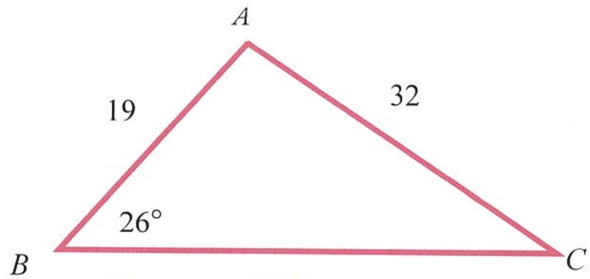
1) (6 points) Solve the following triangles using the appropriate Law. Be sure to show all necessary work. Round answers to nearest whole number:



$180-26-15°$

$A = \boxed{139°}$ $\quad a = \boxed{48}$

$B = 26°$ $\quad b = 32$

$C = \boxed{15°}$ $\quad c = 19$

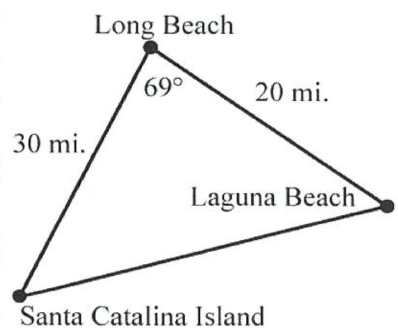
$\frac{\sin C}{19} = \frac{\sin 26°}{32}$

$\Rightarrow C = \sin^{-1}\left(\frac{19 \sin 26°}{32}\right) \approx \boxed{15°}$

$\frac{a}{\sin 139°} = \frac{32}{\sin 26°} \Rightarrow a = \frac{32 \sin 139°}{\sin 26°}$

≈ 48

2) (4 points) Using the illustration to the right, how far is Santa Catalina Island from Laguna Beach? Round to the nearest whole mile.



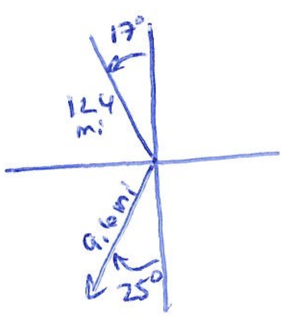
$\sqrt{30^2+20^2-2(30)(20)\cos 69°}$

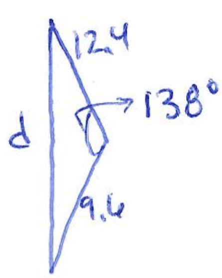
$\approx \boxed{29 \text{ miles}}$

3) (2 points each) Concerning the given information of a triangle, how do you know when to use the Law of Sines versus the Law of Cosines?

I consult the clown that lives under my bed.

4) (7 points) Mike announces a test and two students begin to run away from Mike from the same point. One student runs with a bearing of $S25°W$ at 4.8 mph while the other student runs with a bearing of $N17°W$ at 6.2 mph. How far are the students from each other after 2 hours? Round answer to two decimal places. Draw a picture for this scenario.





$d = \sqrt{12.4^2+9.6^2-2(12.4)(9.6)\cos 138°}$

$\approx \boxed{20.56 \text{ mi}}$

19

5) (4 points each) Convert…

a) $7\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$ to standard form:

$= 7\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= \boxed{\frac{7}{2}-\frac{7\sqrt{3}}{2}i}$

b) $-6\sqrt{3}-6i$ to trigonometric form:

$r=\sqrt{(-6\sqrt{3})^2+(-6)^2} = 12$

$\left.\begin{array}{l}\cos\theta = \frac{-6\sqrt{3}}{12} = -\frac{\sqrt{3}}{2} \\ \sin\theta = -\frac{6}{12} = -\frac{1}{2}\end{array}\right\} \theta = \frac{7\pi}{6}$

$12\left(\cos\left(\frac{7\pi}{6}\right)+i\sin\left(\frac{7\pi}{6}\right)\right)$ or use $210°$

6) (4 points each) For the complex numbers $z_1=\frac{7}{2}-\frac{7\sqrt{3}}{2}i$ and $z_2=-6\sqrt{3}-6i$, find the following using the trigonometric forms and the formula $z_1\times z_2=r_1\times r_2\left[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right]$ for part *a* and $\frac{z_1}{z_2}=\frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right]$ for part *b*. Write in standard form.

a) $z_1\times z_2$

$= 7\cdot 12\left(\cos\left(\frac{5\pi}{3}+\frac{7\pi}{6}\right)+i\sin\left(\frac{5\pi}{3}+\frac{7\pi}{6}\right)\right)$

$= 84\left(\cos\frac{17\pi}{6}+i\sin\frac{17\pi}{6}\right) = 84\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

$\frac{17\pi}{6}$ is coterminal to $\frac{5\pi}{6}$

$= 84\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

$= \boxed{-42\sqrt{3}+42i}$

b) $\frac{z_1}{z_2}$

$= \frac{7}{12}\left(\cos\left(\frac{5\pi}{3}-\frac{7\pi}{6}\right)+i\sin\left(\frac{5\pi}{3}-\frac{7\pi}{6}\right)\right)$

$= \frac{7}{12}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

$= \frac{7}{12}(0+1i) = \boxed{\frac{7}{12}i}$

7) (5 points part *a*; 8 points part *b*) For the complex number $-8+8\sqrt{3}i=16(\cos 120°+i\sin 120°)$, find the following algebraically. For part *a*, use the formula $(a+bi)^n=r^n\left[\cos(n\theta)+i\sin(n\theta)\right]$. For part *b*, use the formula $(a+bi)^{\frac{1}{n}}=r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n}+\frac{360°}{n}\cdot k\right)+i\sin\left(\frac{\theta}{n}+\frac{360°}{n}\cdot k\right)\right]$. Write answers in standard form.

a) $\left(-8+8\sqrt{3}i\right)^2$

$= 16^2(\cos(2\cdot 120°)+i\sin(2\cdot 120°))$

$= 256(\cos 240°+i\sin 240°)$

$= 256\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$= \boxed{-128-128\sqrt{3}i}$

b) The fourth roots of $-8+8\sqrt{3}i$

$16^{\frac{1}{4}}\left(\cos\left(\frac{120°}{4}+\frac{360°}{4}k\right)+i\sin\left(\frac{120°}{4}+\frac{360°}{4}k\right)\right)$

$= 2(\cos(30°+90°k)+i\sin(30°+90°k))$

$k=0$ $2(\cos 30°+i\sin 30°) = \boxed{\sqrt{3}+i}$

$k=1$ $2(\cos 120°+i\sin 120°) = \boxed{-1+\sqrt{3}i}$

$k=2$ $2(\cos 210°+i\sin 210°) = \boxed{-\sqrt{3}-i}$

$k=3$ $2(\cos 300°+i\sin 300°) = \boxed{1-\sqrt{3}i}$

39

8) (3 points) For the point $\left(4, \frac{3\pi}{2}\right)$, find a different representation of the point in polar form that satisfies the following conditions:

a) $r < 0$ and $\theta > 0$

$\boxed{\left(-4, \frac{\pi}{2}\right)}$

b) $r > 0$ and $\theta < 0$

$\boxed{\left(4, -\frac{\pi}{2}\right)}$

c) $r > 0$ and $\theta > 0$

$\boxed{\left(4, \frac{7\pi}{2}\right)}$

9) (3 points) Convert the rectangular point $(-5,-5)$ to the polar format (r,θ):

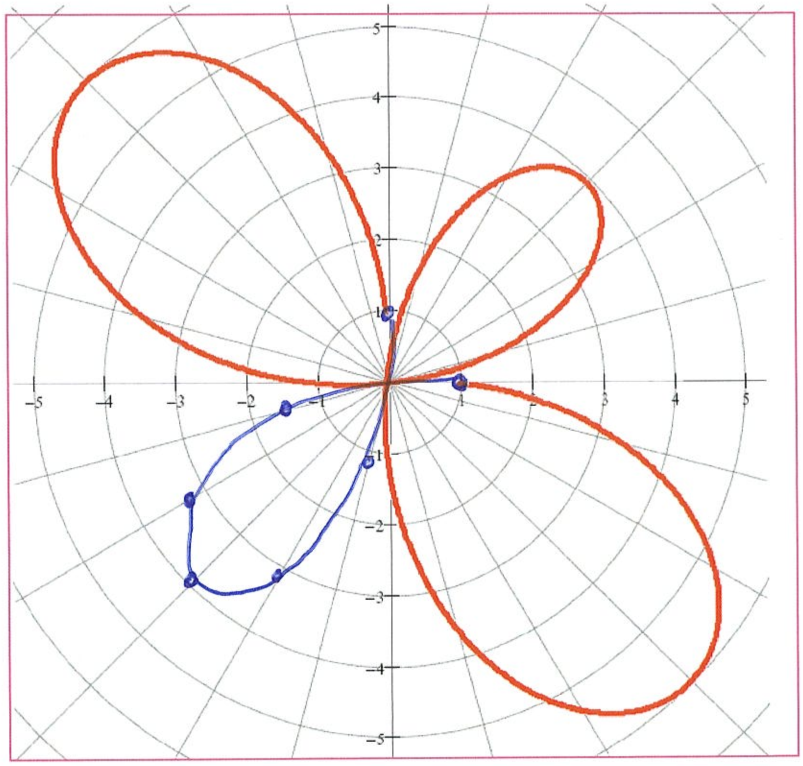
$r = \sqrt{(-5)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$

$\left.\begin{array}{l} \cos\theta = \frac{x}{r} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2} \\ \sin\theta = \frac{y}{r} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{array}\right\} \theta = \frac{5\pi}{4} \text{ or } 225°$

$\boxed{\left(5\sqrt{2}, \frac{5\pi}{4}\right) \text{ or } 225°}$

10) (5 points) Finish the **polar graph** of $r = 1 - 5\sin(2\theta)$ by filling in the chart and plotting points. Round answers to one decimal place:

θ	r
0°	1
15°	-1.5
30°	-3.3
45°	-4
60°	-3.3
75°	-1.5
90°	1



pretty!

17

2

11) (4 points) For the given vectors, determine algebraically if they are equivalent.

$\vec{u}$ has initial point at $(6,-3)$ and terminal point at $(9,5)$

$\vec{v}$ has initial point at $(-4,3)$ and terminal point at $(-7,-5)$

magnitude

$\|\vec{u}\| = \sqrt{(9-6)^2+(5-(-3))^2} = \sqrt{73}$

$\|\vec{v}\| = \sqrt{(-7-(-4))^2+(-5-3)^2} = \sqrt{73}$

direction

$m_{\vec{u}} = \frac{5-(-3)}{9-6} = \frac{8}{3}$

$m_{\vec{v}} = \frac{-5-3}{-7-(-4)} = \frac{8}{3}$

Nope!

or component form

$\vec{u} = \langle 9-6, 5-(-3)\rangle = \langle 3,8\rangle$

$\vec{v} = \langle -7-(-4), -5-3\rangle = \langle -3,-8\rangle$

nope!

12) (3 points each) Let $\vec{u} = \langle 3,-2\rangle$ and $\vec{v} = \langle -1,1\rangle$. Find and simplify:

a) $2\vec{u} - 2\vec{v}$

$\langle 6,-4\rangle + \langle 2,-2\rangle$

$= \boxed{\langle 8,-6\rangle}$

b) $|2\vec{u} - 2\vec{v}|$

$= \sqrt{8^2+(-6)^2}$

$= 10$

c) The unit vector in the same direction as $2\vec{u} - 2\vec{v}$:

$= \frac{1}{10}\langle 8,-6\rangle$

$= \langle \frac{4}{5}, -\frac{3}{5}\rangle$

d) $\vec{u}\cdot\vec{v}$:

$= 3(-1) + (-2)(1)$

$= \boxed{-5}$

e) The angle between the vectors $\vec{u}$ and $\vec{v}$. Round to two decimal places. Use the formula $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$.

$\sqrt{(-1)^2+1^2} = \sqrt{2}$

$\sqrt{3^2+(-2)^2} = \sqrt{13}$

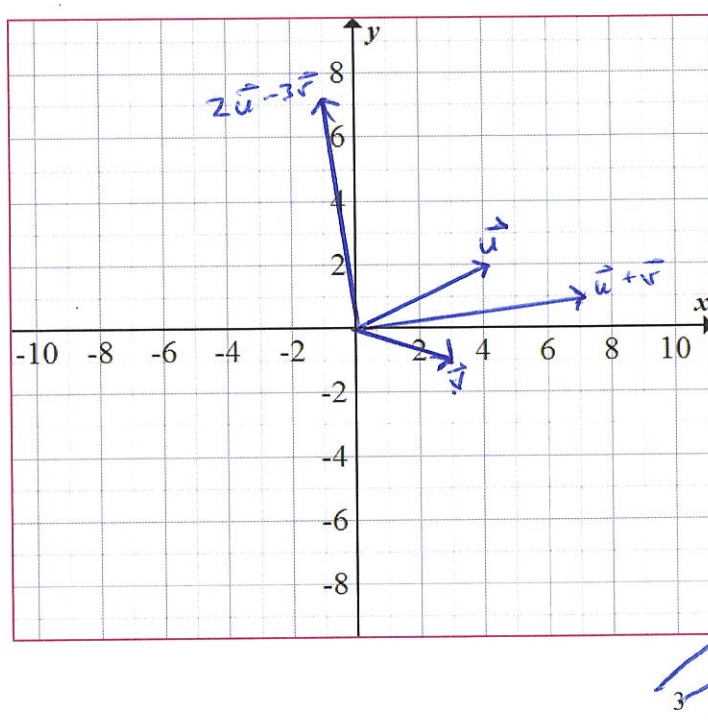
$\cos\theta = \frac{-5}{\sqrt{13}\cdot\sqrt{2}}$

$\theta = \cos^{-1}\left(\frac{-5}{\sqrt{13}\cdot\sqrt{2}}\right) \approx \boxed{168.69°}$

13) (4 points) Given the vector $\vec{u} = \langle 4,2\rangle$ and $\vec{v} = \langle 3,-1\rangle$, draw the vectors $\vec{u}, \vec{v},\ \vec{u}+\vec{v}$, and $2\vec{u} - 3\vec{v}$:

$\vec{u} + \vec{v} = \langle 7,1\rangle$

$2\vec{u} - 3\vec{v} = \langle -1,7\rangle$



23

14) (8 points) Link is traveling in a boat on a bearing of $N28°E$ from Nintendo Island to his new home of Sony Land. His boat travels at a rate of 20 mph. The water has a current from the west at a rate of 3 mph. After an hour, how far is Link from Nintendo Island? Also determine the bearing of the boat from the island. Round **only the final answer** to two places using the formula $\vec{v}=|\vec{v}|(\cos\theta\vec{i}+\sin\theta\vec{j})$.

$\vec{b} = 20(\cos 62°\vec{i} + \sin 62°\vec{j})$

$\vec{w} = 3(\cos 0°\vec{i} + \sin 0°\vec{j})$

$\vec{b}+\vec{w} = \underbrace{(20\cos 62° + 3\cos 0°)}_{A}\vec{i} + \underbrace{(20\sin 62° + 3\sin 0°)}_{B}\vec{j}$

$|\vec{b}+\vec{w}| = \sqrt{A^2+B^2} \approx \boxed{21.57 \text{ miles}}$

$\theta = \tan^{-1}\left(\frac{A}{B}\right) = 35.05°$ $\quad \boxed{N\,35.05°E}$

15) (4 points) Stefani pulls a wagon full of aluminum cans to recycle at a force of 20 pounds on the wagon's handle which makes an angle of 32° to the horizontal. See the figure below. How much work is done pulling the wagon 150 feet? Use the formula $\cos\theta=\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$ and recall that work $=\vec{F}\bullet\overrightarrow{AB}$.

$\vec{F}\cdot\overrightarrow{AB} = |\vec{F}||\overrightarrow{AB}|\cos\theta$

$= \underset{\text{lb}}{20} \cdot \underset{\text{ft}}{150} \cdot \cos 32°$

$= 2544.14$ ft-lb.

12