1) (8 points) Fill in the information for the parabola $(x-4)^2 = \overset{4p}{\overset{\frown}{8}}(y+1)$:

i) $h =$ ___4___

ii) $k =$ ___-1___

iii) $p =$ ___2___

Write iv and v as ordered pairs:

iv) Vertex ___$(4,-1)$___

v) Focus ___$(4,1)$___

vi) Directrix ___$y = -3$___

vi) Sketch the graph:

$y = -3$
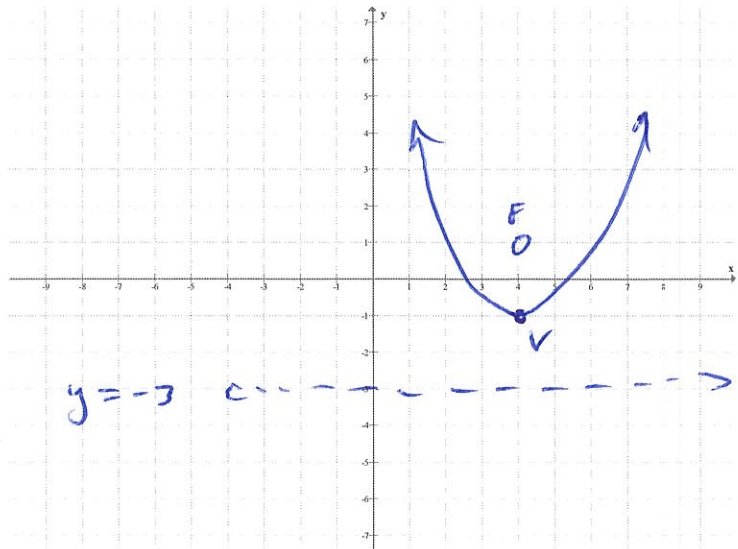
2) (4 points) While on stage, Mike notices that the spotlight on him has a parabolic mirror that is 4 feet wide and 1.5 feet deep. If the bulb for the spotlight is at the focus of the parabola, how far, in inches, from the vertex of the parabola is the bulb located?


4ft · (2, 1.5) · 1.5 ft
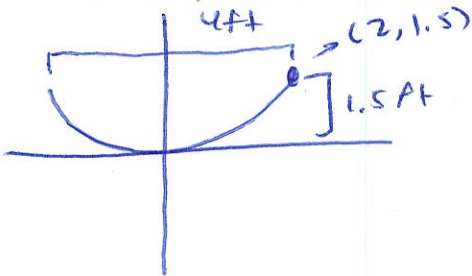
$x^2 = 4py$

$2^2 = 4p(1.5) \Rightarrow \boxed{p = \frac{2}{3} \text{ft}}$

3) (3 points each) Statuary Hall, also known as the Whispering Gallery, is an elliptical room in the United States Capitol in Washington D.C. where a person standing at one focus of the room can hear even a whisper spoken by a person standing at the other focus. Statuary Hall is 46 feet wide and 97 feet long. Assuming a horizontal ellipse, find…

a) The equation of the ellipse for the room:

$\boxed{\dfrac{x^2}{\left(\frac{97}{2}\right)^2} + \dfrac{y^2}{23^2} = 1}$

46
b=23   d
2352.25   529

97    $a = \frac{97}{2}$

b) The location of the foci from the center of the room. Round to one decimal place:
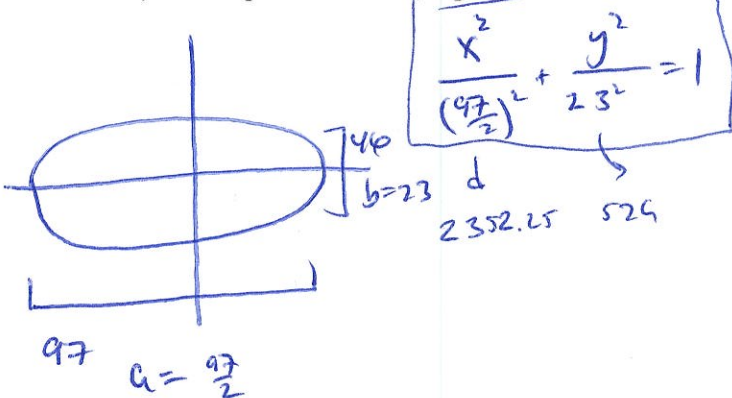
$c^2 = a^2 - b^2 = \left(\frac{97}{2}\right)^2 - 23^2$

$c^2 = 1823.25$

$\boxed{c \approx 42.7 \text{ft}}$

2

4) (8 points) Fill in the information for the hyperbola $\dfrac{y^2}{25} - \dfrac{x^2}{144} = 1$:

i) $h =$ ___0___

ii) $k =$ ___6___

iii) $a =$ ___5___

iv) $b =$ ___12___

v) $c =$ ___13___

Write as ordered pairs:

vi)   Center ___(0, 0)___

vii) Vertices ___(0, -5) (0, 5)___

viii) Co-Vertices ___(12, 0) (-12, 0)___

ix)   Foci ___(0, 13) (0, -13)___

Write in slope-intercept form:

x)   Asymptotes ___$y = 5/12\,x$___

$y = -5/12\,x$

vi) Sketch the graph:

$y = \frac{5}{12}x$

$y = -\frac{5}{12}x$
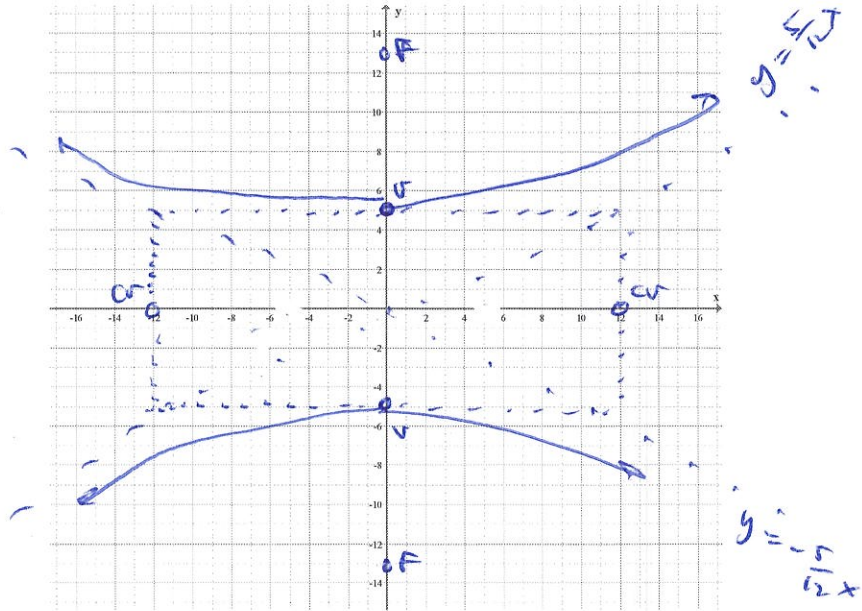
5) (6 points each) Solve the following systems. For part *a*, write your answer in ordered pairs. For part *b*, shade your final in the darkest:

a) $\begin{cases} 4x^2 + y^2 = 16 \\ x^2 - y^2 = 4 \end{cases}$

b) $\begin{cases} x^2 + y^2 \le 9 \\ \dfrac{x^2}{25} + \dfrac{y^2}{4} \le 1 \end{cases}$

$5x^2 = 20$

$x^2 = 4$

$x = 2$ $x = -2$

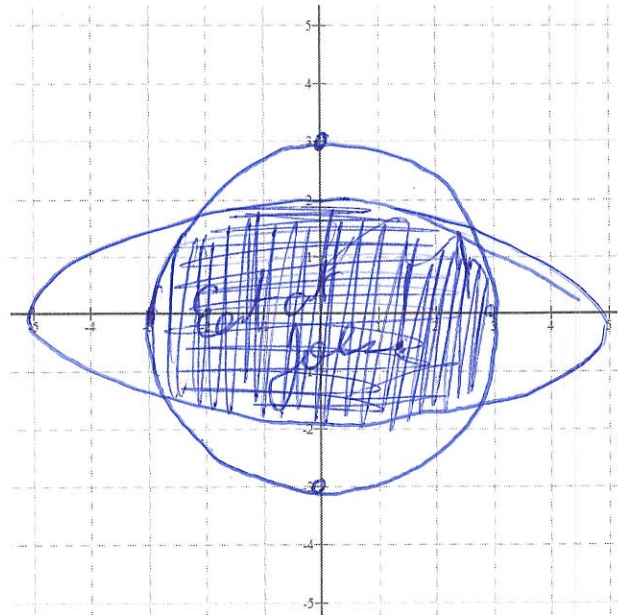$2^2 - y^2 = 4$ $(-2)^2 - y^2 = 4$

$-y^2 = 0$ $-y^2 = 0$

$y = 0$ $y = 0$

$(2, 0)$ $(-2, 0)$

3

6) (2 points) What is a sequence?

beep boop boop boop beep beep boop boop boop boop beep ping!

7) (6 points each) Find the first five terms of the following sequences. Determine if they are arithmetic, geometric, or neither. If it is arithmetic, determine the common difference. If it is geometric, determine the common ratio.

a) $\{2(-3)^n\}$

$-6, 18, -54, 162, -486$

geometric

$r = -3$

b) $a_1 = 3, a_{n+1} = a_n + \frac{1}{2}, n \geq 1$

$3, 3.5, 4, 4.5, 5$

arithmetic

$d = \frac{1}{2}$

8) (4 points) Find the sum $\sum_{k=1}^{5} \frac{k}{3}$. Be sure to write out the individual terms:

$= \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \frac{4}{3} + \frac{5}{3} = \frac{15}{3} = $ ⑤

9) (4 points) Write in sigma notation: $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + ... + \frac{49}{50}$

$\sum_{k=1}^{49} \frac{k}{k+1}$

10) (3 points each) For the arithmetic sequence $-4, -1, 2, 5, 8...$, find and simplify...

a) a_n using $a_n = a_1 + (n-1)d$
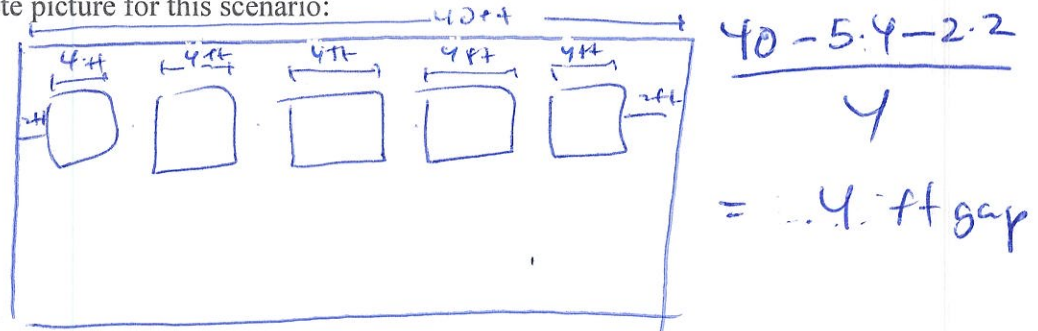
$a_n = -4 + (n-1) \cdot 3$

$a_n = 3n - 7$

b) a_{412}:

$a_{412} = 3(412) - 7$

$a_{412} = 1229$

4

11) (4 points each) Mike decides to hang some photos of his favorite pies on a 40 foot wall in his office. The 5 frames are 4 feet long each. He wants the first and last frames to be in from the corner by 2 feet and equal spacing in between each frame.

a) Draw a complete picture for this scenario:

$$\frac{40 - 5 \cdot 4 - 2 \cdot 2}{4}$$

$$= 4 \text{ ft gap}$$

b) Assuming the nails will go in the exact horizontal center of the frame, where should Mike put the nails into the wall? Start your count from the left corner:

$2 + 4 \div 2$

4 ft, 12 ft, 20 ft, 28 ft, 36 ft
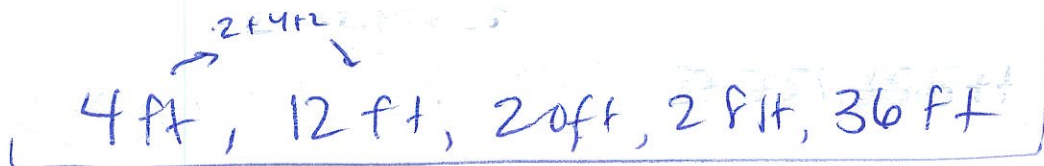
12) (4 points) Evaluate the sum $\sum_{k=12}^{48}\left(\dfrac{5k+10}{4}\right)$ using either formula: $S_n = \dfrac{n}{2}(2a_1 + (n-1)d)$ or

$S_n = \dfrac{n}{2}(a_1 + a_n)$. Be sure to show the setup of the formula you pick.

$48 - 12 + 1$

$= 37$

$\hookrightarrow S_{37} = \dfrac{37}{2}\left(\overset{k=12}{\dfrac{35}{2}} + \overset{k=48}{\dfrac{125}{2}}\right) = \boxed{1480}$

13) (3 points each) For the geometric sequence $\dfrac{2}{27}, \dfrac{2}{9}, \dfrac{2}{3}, 2, ...$, find the following given the general

term of a geometric sequence is $a_n = a_1 r^{n-1}$:

a) a_n

$a_n = \left(\dfrac{2}{27}\right)(3)^{n-1}$

b) a_{12}

$a_{12} = \left(\dfrac{2}{27}\right)(3)^{12-1} = \boxed{13{,}122}$

5

$$\frac{1}{64} = P\left(\frac{1}{2}\right)^{n-1}$$
$$n = 10$$

14) (5 points each) Using the formulas $S_n = a_1 \dfrac{1-r^n}{1-r}$ and $S_\infty = \dfrac{a_1}{1-r}$ (respectively), find by writing answers as an improper fraction (as needed) …
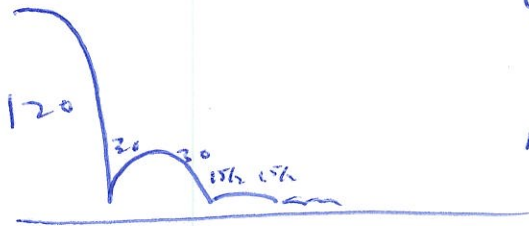
a) $8+4+2+1+...+\dfrac{1}{64}$

$a_1 = 8 \quad r = \frac{1}{2}$

$\curvearrowright) \; n = 10$

$S_{10} = 8 \cdot \dfrac{1-\left(\frac{1}{2}\right)^{10}}{1-\frac{1}{2}} = \boxed{\dfrac{1023}{64}}$

b) $8+4+2+1+...$

$\hookrightarrow = \dfrac{8}{1-\frac{1}{2}} = \boxed{16}$

15) (6 points) A ball is dropped from a height of 120 feet and always rebounds $\frac{1}{4}$ of the distance fallen. How far does the ball travel vertically before coming to a stop? In your answer be sure to use the formula $S_\infty = \dfrac{a_1}{1-r}$.

drop $\quad \dfrac{120}{1-\frac{1}{4}}$

$+$

rebound $\quad \dfrac{30}{1-\frac{1}{4}}$

$\boxed{200 \, ft}$

T6