1) (3 points each) Find the inverse of the following functions:

a) $\{(6,-2),(-5,8),(0,-9),(16,1)\}$

$\{(-2,6),(8,-5),(-9,0),(1,16)\}$

b) $f(x)=\dfrac{x+7}{x-5}$

① $y=\dfrac{x+7}{x-5}$

② $x=\dfrac{y+7}{y-5}$

③ $xy-5x=y+7$

$xy-y=5x+7$

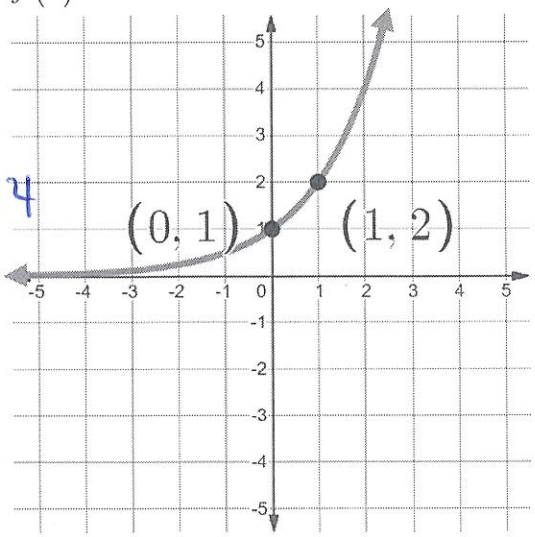
$y(x-1)=5x+7$

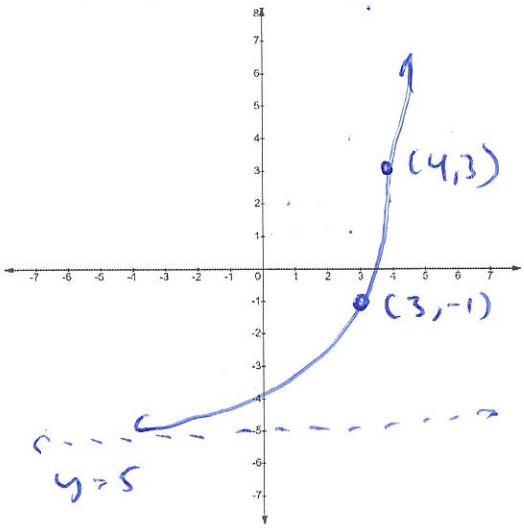
$y=\dfrac{5x+7}{x-1}$

④ $f^{-1}(x)=\dfrac{5x+7}{x-1}$

2) (3 points) Using the graph of $y=2^x$ shown on the left, sketch the graph of the function $f(x)=4\cdot 2^{x-3}-5$. Be sure to label the transformed given points and asymptote:

1) Right 3
2) Vert. stretch by 4
3) Down 5





3) (3 points) Algebraically find the inverse of the function $f(x)=4\cdot 2^{x-3}-5$:

① $y=4\cdot 2^{x-3}-5$

② $x=4\cdot 2^{y-3}-5$

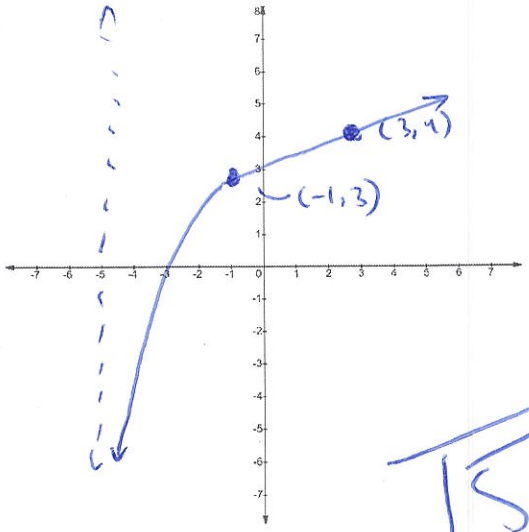
③ $\dfrac{x+5}{4}=2^{y-3}$

$\log_2\left(\dfrac{x+5}{4}\right)=y-3$

$y=\log_2\left(\dfrac{x+5}{4}\right)+3$

④ $f^{-1}(x)=\log_2\left(\dfrac{x+5}{4}\right)+3$

4) (3 points) Sketch a graph of the inverse of the function $f(x)=4\cdot 2^{x-3}-5$ Be sure to label the transformed given points and asymptote. *Hint, use your answer from #2 and remember the relationship between the graph of a function and the graph of its inverse.*



15

5) (4 points) Write the expression as one logarithm: $5\log x + 9\log y - \frac{1}{3}\log z$

$= \log x^5 + \log y^9 - \log z^{1/3} = \log \frac{x^5 y^9}{z^{1/3}}$

6) (4 points) Given that $\log_a x = 3$, $\log_a y = -4$, and $\log_a z = 5$, find the value of $\log_a \sqrt{\frac{x^3 y^5}{z^2}}$:

$= \frac{1}{2}\log\left(\frac{x^3 y^5}{z^2}\right) = \frac{1}{2}\left[\log x^3 + \log y^5 - \log z^2\right]$

$= \frac{1}{2}\left[3\log x + 5\log y - 2\log z\right] = \frac{1}{2}\left[3\cdot 3 + 5(-4) - 2(5)\right]$

$= -\frac{21}{2}$

7) (3 points each) Simplify completely:

a) $\log_{25}(5x)$

$= \log_{25} 5 + \log_{25} x$

$= 1/2 + \log_{25} x$

b) $\ln\left(\frac{e^9}{x^2}\right)$

$= \ln e^9 - \ln x^2$

$= 9 - 2\ln x$

c) $\ln e^{-5} + \log_2 8$

$= -5 + 3 = -2$

8) (5 points each) Solve for the variable.

a) $4^{3x+7} = 64$

$4^{3x+7} = 4^3$

$3x + 7 = 3$

$3x = -4$

$x = -\frac{4}{3}$

b) $5^{3x-2} = 35$

$\log_5 35 = 3x - 2$

$\frac{\log_5 35 + 2}{3} = x$

c) $\ln(4x-7) - \ln(x-2) = \ln(5)$

$\ln\left(\frac{4x-7}{x-2}\right) = \ln 5$

$\frac{4x-7}{x-2} = 5$

$4x - 7 = 5x - 10$

$3 = x$

d) $\log_2(5x+15) - \log_2(x+2) = 3$

$\log_2 \frac{5x+15}{x+2} = 3$

$8 = 2^3 = \frac{5x+15}{x+2}$

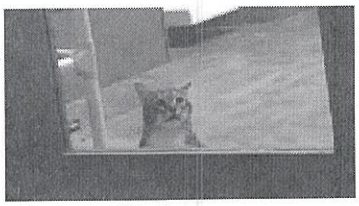
$8x + 16 = 5x + 15$

$3x = -1$

$x = -1/3$

37

9) (4 points each) The video "Shelter Cat Plans World Domination" started to go viral last month. At 8 am, when the video was posted, there were 300 views. At noon, there 15,000 views. Assume that the number of views is growing exponential and use the formula $P(t) = P_0 e^{kt}$ where P is the number of views and t is the number of hours past 8 am.



a) Determine the **exact** value for the growth rate k.

$15000 = 300e^{k(4)}$

$50 = e^{4k}$

$\ln 50 = 4k$

$k = \frac{\ln 50}{4}$

b) How many hours past 8 am will there will 350,000 views? Round your answer to one decimal place.

$P(t) = 300e^{\frac{\ln 50}{4}t}$

$350000 = 300e^{\frac{\ln 50}{4}t}$

$\frac{3500}{3} = e^{\frac{\ln 50}{4}t}$

$\ln\frac{3500}{3} = \frac{\ln 50}{4}t$

$t = \frac{\ln\frac{3500}{3}}{\frac{\ln 50}{4}}$

$t = 7.2$ hrs

10) (1 measly point) Fill in the blank: John Jacob Jingleheimer Schmidt, a foreign exchange student from Norway, is in your math class. (Yes, in the future, you'll be teaching math—kudos: me.) He asks one day for you to pronounce ln12 for him. You reply "Gladly, it's pronounced natural log of twelve"

11) (9 points each) Solve the system $\begin{cases} x - 7y + 5z = 3 \\ -3x - y + 3z = 25 \\ 4x - 2y + 3z = 6 \end{cases}$ using the methods listed below. Be sure to show all necessary work where appropriate. Write answer as an ordered triple:

a) Elimination Method:

① $3x - 21y + 15z = 9$
$-3x - y + 3z = 25$
$-22y + 18z = 34$

② $-4x + 28y - 20z = -12$
$4x - 2y + 3z = 6$
$26y - 17z = -6$

×22 ×26

③ $-572y + 468z = 884$
$572y - 374z = -132$
$94z = 752$

④ $z = 8$

⑤ $-22y + 18(8) = 34$
$-22y = -110$
$y = 5$

$x - 7(5) + 5(8) = 3$
$x = -2$

$(-2, 5, 8)$

b) Gauss-Jordan Method:

$$\left[\begin{array}{ccc|c} 1 & -7 & 5 & 3 \\ -3 & -1 & 3 & 25 \\ 4 & -2 & 3 & 6 \end{array}\right] \xrightarrow{\substack{3R_1+R_2\to R_2 \\ -4R_1+R_3\to R_3}} \left[\begin{array}{ccc|c} 1 & -7 & 5 & 3 \\ 0 & -22 & 18 & 34 \\ 0 & 26 & -17 & -6 \end{array}\right] \xrightarrow{\substack{-\frac{1}{22}R_2\to R_2 \\ 7R_2+R_1\to R_1 \\ -26R_2+R_3\to R_3}} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{8}{11} & -\frac{86}{11} \\ 0 & 1 & -\frac{9}{11} & -\frac{17}{11} \\ 0 & 0 & \frac{47}{11} & \frac{376}{11} \end{array}\right] \xrightarrow{\substack{\frac{11}{47}R_3\to R_3 \\ \frac{8}{11}R_3+R_1\to R_1 \\ \frac{9}{11}R_3+R_2\to R_2}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$(-2, 5, 8)$

12) (1 point) Verify that you made absolutely sure that your answer to 11a is the same as in 11b by signing your name here Ohmygodtherearefractions. You will not receive the credit if the work does not support the same answer.

13) For the following problem:

In a certain country people own a total of about 353 million fish, cats, and dogs as pets. The number of fish owned is 7 million more than the total number of cats and dogs owned, and 16 million more cats are owned than dogs. How many of each type of pet do people in this country own?

a) (3 points) Name and define your variables for this problem:

x = # of fish
y = # of cat
z = # of dogs

b) (5 points) Set up **BUT DO NOT SOLVE** a system of equations for this problem:

$$\begin{cases} x+y+z=353 \\ x=7+y+z \\ y=16+z \end{cases} \quad \text{— or —} \quad \begin{cases} x+y+z=353 \\ x-y-z=7 \\ y-z=16 \end{cases}$$

14) (6 points each) Decompose into partial fractions. Be sure to show the system you are using to solve for the necessary letters.

a) $\dfrac{x+10}{x^2-4x-12} = \dfrac{x+10}{(x-6)(x+2)}$

$\dfrac{x+10}{(x-6)(x+2)} = \dfrac{A}{x-6} + \dfrac{B}{x+2} = \dfrac{2}{x-6} + \dfrac{-1}{x+2}$

$x+10 = A(x+2) + B(x-6)$

$x+10 = Ax+2A+Bx-6B$

$\begin{cases} A+B=1 \\ 2A-6B=10 \end{cases} \Rightarrow \begin{matrix} A=2 \\ B=-1 \end{matrix}$

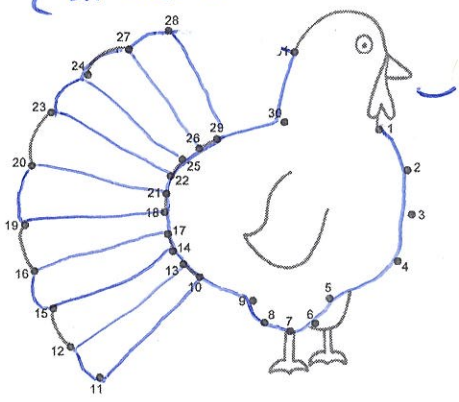
b) $\dfrac{x^2+2x+7}{(x^2+2)(x+1)} = \dfrac{Ax+B}{x^2+2} + \dfrac{C}{x+1}$

$x^2+2x+7 = (Ax+B)(x+1) + C(x^2+2)$

$= Ax^2+Ax+Bx+B+Cx^2+2C$

$\begin{cases} A+C=1 \\ A+B=2 \\ B+2C=7 \end{cases} \Rightarrow \begin{matrix} A=-1 \\ B=3 \\ C=2 \end{matrix}$

$\dfrac{-x+3}{x^2+2} + \dfrac{2}{x+1}$



21