1) (3 points) Solve for the variable in $x^4 - x^2 - 2 = 0$ by rewriting the equation as a quadratic.

Let $u = x^2$

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2 \qquad u = -1$

$x^2 = 2 \qquad x^2 = -1$

$$\boxed{x = \pm\sqrt{2} \qquad x = \pm i}$$

2) (2 points each) For the function $f(x) = 4x^2 - 12x + 1$, determine…

a) If it opens up or down. How do you know?

up $a = 4 > 0$

b) The coordinates of the vertex (by hand)

$x = -\frac{(-12)}{2(4)} = \frac{3}{2}$

$f(3/2) = -8$

$\left(\frac{3}{2}, -8\right)$

c) The domain

$\mathbb{R}$

d) The range

$[-8, \infty)$

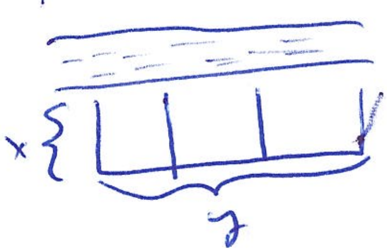
e) Interval of increase

$(3/2, \infty)$

f) Interval of decrease

$(-\infty, 3/2)$

3) (6 points) Forza is building some gardens on his lovely island of Pixel Cove. He wants to build 3 adjacent rectangular pens which will border a river to showcase his prize flowers. The side against the river will not receive any fencing. He has 192 feet of fencing available. What should the dimensions be of the enclosure to maximize the area? What is the maximum area?



$4x + y = 192$

$y = 192 - 4x$

$A(x) = xy = x(192 - 4x)$

$= -4x^2 + 192x$

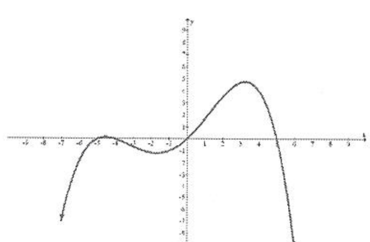
$x = -\frac{192}{2(-4)} = \underline{24}$

$y = 192 - 4(24) = \underline{96}$

$\underline{A = 24 \cdot 96 = 2304 \text{ ft}^2}$

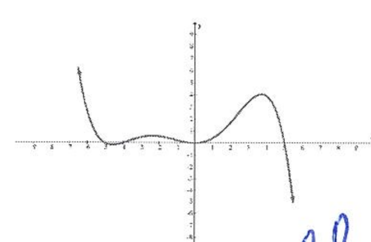
4) (2 points each) Give an example of a function which will have similar arrowheads to the function below:

a)



$f(x) =$ neg x^{even}

b)



$f(x) =$ ~~neg~~ neg x^{odd}

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5) (4 points each) For the function $f(x)=(x+4)^2(2x+1)^2(x-3)\ldots$

a) Find the leading term and state which quadrants the arrowheads will be in and why:

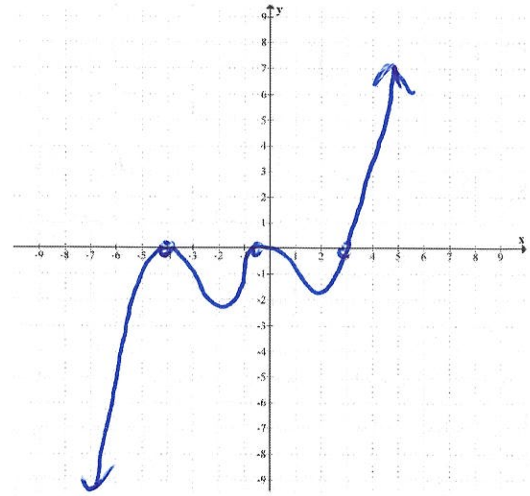
$4x^5$ odd   Q I & III

pos

b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
-4	2	
$-1/2$	2	
3	1	

c) Sketch the graph based on parts *a* and *b*:



6) (3 points each) Form a polynomial function of degree five that meets the following requirements. **Be sure to leave your answer in factored form.**

a) Has zeros including -3, $6+5i$, and 4 is a zero of multiplicity 2

$f(x)=(x+3)(x-(6+5i))(x-(6-5i))(x-4)^2$

b) Has the same zeros and multiplicity as in part *a* but is a different function

$g(x)=6(x+3)(x-(6+5i))(x-(6-5i))\cdot(x-4)^2$

any nonzero #

7) (4 points part *a*, 2 points each *b* and *c*) Consider the functions $f(x)=x^3+4x^2-7x-10$ and $g(x)=x^2-x-2$.

a) Divide $f(x)$ by $g(x)$ using long division.

$$\begin{array}{r} x+5 \\ x^2-x-2 \enclose{longdiv}{x^3+4x^2-7x-10} \\ -(x^3-x^2-2x) \\ \hline 5x^2+5x-10 \\ -(5x^2-5x-10) \\ \hline 0 \end{array}$$

b) Based on your work in part *a*, was $g(x)$ a factor of $f(x)$? Why or why not?

Yup! Remainder was zero

c) What is the equation of the oblique asymptote of the rational function $y=\dfrac{x^3+4x^2-7x-10}{x^2-x-2}$?

$y=x+5$

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8) (8 points) Factor the polynomial completely by first listing the possible rational roots and then using synthetic division and your calculator.

$$f(x) = x^4 - 6x^3 + 10x^2 + 2x - 15$$

$p = \pm 1, 3, 5, 15$

$q = \pm 1$

$\frac{p}{q} = \pm 1, 3, 5, 15$

$$\begin{array}{r|rrrrr} -1 & 1 & -6 & 10 & 2 & -15 \\ & & -1 & 7 & -17 & 15 \\ \hline 3 & 1 & -7 & 17 & -15 & 0 \\ & & 3 & -12 & 15 & \\ \hline & 1 & -4 & 5 & 0 & \end{array}$$

$\hookrightarrow x^2 - 4x + 5$

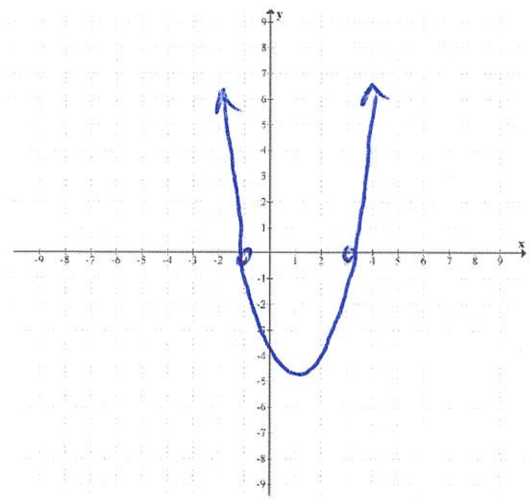
$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$

$x = \frac{4 \pm \sqrt{-4}}{2} = 2 \pm i$

$f(x) = (x+1)(x-3)(x-(2+i))(x-(2-i))$

9) (3 points each) Continuing from number 8, for the function $f(x) = x^4 - 6x^3 + 10x^2 + 2x - 15$, based on your work in number 8, complete the chart and sketch the graph.

Zero	Multiplicity	Touch/Cross
-1	1	cross
3	1	cross



10) (4 points each) For the function $f(x) = \dfrac{x^2 - 3x + 2}{x^3 - x^2 - 2x}$, find…

a) The domain

$x^3 - x^2 - 2x = 0$

$x(x-2)(x+1) = 0$

$x \neq 0, 2, -1$

b) The x- and y-intercepts. Label your answers.

x-int

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$\cancel{x = 2}$ $x = 1$

y-int

$f(0) = \frac{2}{0}$ DNE

NO Y-int

c) Any vertical asymptotes and holes

$x = 0$

$0^2 - 3(0) + 2 \neq 0$

$\boxed{x = 0 \text{ VA}}$

$x = -1$

$(-1)^2 - 3(-1) + 2 \neq 0$

$\boxed{x = -1 \text{ VA}}$

$x = 2$

$2^2 - 3(2) + 2 = 0$

Hole

$\frac{\cancel{(x-2)}(x-1)}{x\cancel{(x-2)}(x+1)} = \frac{x-1}{x(x+1)}$

@ $x = 2$ $\frac{2-1}{2(2+1)} = \frac{1}{6}$ $\boxed{\text{Hole}(2, \tfrac{1}{6})}$

d) Any horizontal or oblique asymptotes

HA $y = 0$

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11) (3 points) Solve for the variable in $x^2+7x+6 \geq 0$. Write your answer in interval notation:

$(x+6)(x+1)=0$

$x=-6,-1$

$+ \quad - \quad +$

$-7 \quad -6 \quad -2 \quad -1 \quad 0$

$\boxed{(-\infty,-6] \cup [-1,\infty)}$

12) (2 points per space) Fill in the blank:

a) If c is a zero of a polynomial function $f(x)$, then $f(c)=$ ______ and ______ is a factor of the function $f(x)$.

b) Numbers not in the domain of a rational function lead to ______________________.

13) (3 points each) Short answer. Clearly explain how to find the following algebraically:

a) Vertical Asymptotes and Holes:

b) Horizontal and Oblique Asymptotes:

14) (1 point per space) Fill in the blank:

a) A polynomial of degree n will have at most ______ x-intercept.

b) The sum of the multiplicities of the zeros of a polynomial function always add up to the ________ of the function.

c) If the multiplicity of a zero is even, the graph ________ the x-axis at that value while if the multiplicity of a zero is odd, the graph ________ the x-axis at that value.

Extra Credit (2 points) : Find the equation of a rational function **in factored form** that has all of the following properties:

a) Hole at $x=3$

b) Vertical Asymptotes at $x=-4$ and $x=2$

c) x-intercepts at $x=\frac{2}{3}$ and $x=6$

d) Horizontal asymptote at $y=3$

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