1) (3 points) Write the expression as a sum or difference of logarithms:

$$\ln\left(\frac{x^3y^2}{z^4}\right)$$

$= \ln(x^3y^2) - \ln z^4 = \ln x^3 + \ln y^2 - \ln z^4$

$= \boxed{3\ln x + 2\ln y - 4\ln z}$

2) (4 points each) Solve for the variable. Be sure to find the exact value.

a) $2^{2x-1} = 32^{2x+4}$

$2^{2x-1} = (2^5)^{2x+4}$

$2^{2x-1} = 2^{10x+20}$

$2x-1 = 10x+20$

$\boxed{x = -\frac{21}{8}}$

b) $5e^{5x-3}+2=13$

$e^{5x-3} = \frac{11}{5}$

$\ln\frac{11}{5} = 5x-3$

$\boxed{x = \frac{\ln 11/5 + 3}{5}}$

c) $\log_2(x+1)+\log_2(x-1)=\log_2 24$

$\log_2(x^2-1) = \log_2 24$

$x^2-1=24$

$x^2=25$

$x=\pm 5$

$\boxed{x=5}$

d) $2\log(3x+4)-3=-1$

$\log(3x+4)=1$

$10^1 = 3x+4$

$\boxed{x=2}$

3) (4 points each) The video "Shelter Cat Plans World Domination" started to go viral last month. At 8 am, when the video was posted, there were 300 views. At noon, there 15,000 views. Assume that the number of views is growing exponential and use the formula $P(t)=P_0e^{kt}$ where P is the number of views and t is the number of hours past 8 am.

a) Determine the exact value for the growth rate k.

$P(t) = 300e^{kt}$

$15000 = 300e^{k\cdot 4}$

$50 = e^{4k}$

$\ln 50 = 4k$

$\boxed{k = \frac{\ln 50}{4}}$

b) How many hours past 8 am will there will 350,000 views? Round your answer to one decimal place.

$350000 = 300e^{\frac{\ln 50}{4}t}$

$\frac{3500}{3} = e^{\frac{\ln 50}{4}t}$

$\ln\frac{3500}{3} = \frac{\ln 50}{4}t$

$t = \frac{\ln(3500/3)}{\ln 50/4} \approx \boxed{7.2 \text{ hrs}}$

2

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4) (3 points) Short answer. In algebra, why do we need to use algorithms?

because...

5) (4 points) If you deposited $10,000 at a simple interest rate of 1.05%, how much would you have after 10 years and how much interest would be earned? Be sure to label your answers.

simple int

$I = 10000(0.0105)(10) =$ \$1050 interest

$A = 10000 + 1050 =$ \$11050 total

6) (4 points) If you deposited $10,000 at an annual interest rate of 1.05% compounded monthly, how much would you have after 10 years and how much interest would be earned? Be sure to label your answers.

compound int.

$A = 10000\left(1 + \frac{0.0105}{12}\right)^{12 \cdot 10} =$ \$11,106.60 total

$11,106.60 - 10,000 =$ \$1106.60 interest

7) (4 points) How many years would be needed for a deposit of $7,000 to grow to $15,000 if it were deposited into an account that offered a 2.17% annual interest rate compounded annually?

Compound int

$15000 = 7000\left(1 + \frac{0.0217}{1}\right)^{1 \cdot t}$

$\frac{15}{7} = (1.0217)^t$

$\log_{1.0217} 15/7 = t$

$t \approx 35.50$ years

8) (4 points) How much should be invested now so that in 12 years there will be $8,300 in an account that offers a 2.35% annual interest rate compounded quarterly?

Present value

$P = 8300\left(1 + \frac{0.0235}{4}\right)^{-4 \cdot 12}$

$= \$6265.64$

9) (4 points) Which is a better way to invest? Option A: 2.35% compounded semi-annually or Option B: 2.27% compounded monthly? **Write answer as a percent rounded to two decimal places.**

APY

A: $\left(1+\frac{0.0235}{2}\right)^{2}-1$

2.36%

Better

B: $\left(1+\frac{0.0227}{12}\right)^{12}-1$

2.29%

10) (4 points each) Igor borrowed $215,000 for a home on a 30-year loan that carried a 6.35% annual interest rate compounded monthly. After 10 years, he was able to refinance down to a 15-year loan that carried a 2.93% annual interest rate compounded monthly.

AMORT

a) Determine the monthly payment for the beginning 30-year loan:

$$PMT=\frac{215000\left(\frac{.0635}{12}\right)}{\left(1-\left(1+\frac{.0635}{12}\right)^{-12\cdot 30}\right)}$$

$=\$1337.81$

Amount owed on a loan

b) How much was left on the balance after paying for 10 years?

$$\frac{215000\left(1+\frac{.0635}{12}\right)^{12\cdot 10}-1337.81\left(\left(1+\frac{.0635}{12}\right)^{12\cdot 10}-1\right)}{\left(\frac{.0635}{12}\right)}$$

$=\$181,577.08$

AMORT

c) Determine the monthly payment for the new 15-year loan:

$$PMT=\frac{181577.08\left(\frac{.0293}{12}\right)}{\left(1-\left(1+\frac{.0293}{12}\right)^{-12\cdot 15}\right)}$$

$=1247.23$

d) How much money did Igor save by refinancing his mortgage?

$1337.81\cdot 12\cdot 20$

$-1247.23\cdot 12\cdot 15$

$\$96,465$

11) (5 points each) Romesh works out that he would need $6,200 a month during his retired years. He is currently 25 years old and plans to work until his is 65. He assumes that he would need to make withdraws for 30 years past his retirement. Assuming he finds an account that will offer him a 6.25% annual interest rate compounded monthly for the entire duration of the account…

PV, A

a) How much should he have in his account at retirement?

$$PV=\frac{6200\left(1-\left(1+\frac{.0625}{12}\right)^{-12\cdot 30}\right)}{\left(\frac{.0625}{12}\right)}$$

$=\$1,006,955.79$

Sinking fund

b) How much should he deposit monthly during his working years to ensure he meets his goal?

$$PMT=\frac{1006955.79\left(\frac{.0625}{12}\right)}{\left(\left(1+\frac{.0625}{12}\right)^{12\cdot 40}-1\right)}$$

$=\$472.33$

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12) Katherine is 30 years old and is setting up her retirement account. She is able to deposit $525 a month a 401-k which offers a 7.25% annual interest rate. She does this for 30 years. After that time, she will retire. She wishes, over the next 25 years, to take out equal withdraws until the account is emptied. Assume the interest rate is the same after retirement.

a) (9 points) What are the equal withdraws she is able to take out?

FV

$FV = \frac{525\left(\left(1+\frac{0.0725}{12}\right)^{12\cdot 30}-1\right)}{\left(\frac{.0725}{12}\right)}$

$= \$650560.84 \cdot 672993.98$

t

Amort

$PMT = \frac{650,560.84\left(\frac{.0725}{12}\right)}{\left(1-\left(1+\frac{.0725}{12}\right)^{-12\cdot 25}\right)}$

$= \$4702.30 \cdot 4864.45$

b) (2 points) How much did she deposit before retirement?

$\$525 \cdot 12 \cdot 30$

$= \$189,000$

c) (2 points) How much did she withdraw after retirement?

$\$4702.30 \cdot 12 \cdot 25$

$= \$1,459375.00$

d) (2 points) How much interest did she earn overall?

$\$1459375.00 - 189000$

$= \$1,270,375$

13) Hope takes finds a home for $170,000. He plans to put down 25% and finance the rest at a 3.15% annual interest rate for 30 years. He determines his monthly mortgage payment to be $547.91.

a) (6 points) Chart the payment breakdown for the first two months of Hope's home. Be sure to show all necessary work.

Interest	Principal	Balance
$I = 127500(.0315 \times \frac{1}{12})$ $I = 334.69$	$547.91 - 334.69$ $= 213.22$	$127500 - 213.22$ $= \$127,286.78$
$I = 127,286.78(.0315)(\frac{1}{12})$ $= 334.13$	$547.91 - 334.13$ $= 213.78$	$127,286.78 - 213.78$ $= \$127,073.00$

b) (3 points) How much total interest did Hope pay in interest for his home?

$547.91 \cdot 12 \cdot 30 - \underbrace{127500}_{\text{loan amount}} = \$69,747.60$

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