1) (3 points) Solve for the variable in $x^4 - 1 = 0$. *Hint, there are four answers.*

let $u = x^2$ $\quad u^2 - 1 = 0$

$(u+1)(u-1) = 0$

$u = 1$ $\quad x^2 = 1$ $\quad x = \pm 1$

$u = -1$ $\quad x^2 = -1$ $\quad x = \pm i$

2) (2 points each) For the function $f(x) = 2x^2 - 8x + 1$, determine algebraically…

a) If it opens up or down. How do you know?

up $a = 2 > 0$

b) The coordinates of the vertex:

$x = -b/2a = \frac{(-8)}{2(2)} = 2$

$f(2) = -7$

$(2, -7)$

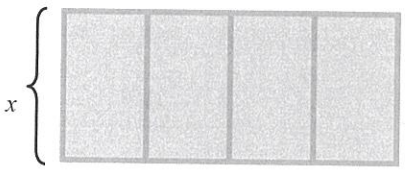
c) The domain: $\mathbb{R}$

d) The range: $[-7, \infty)$

e) Interval of increase: $(2, \infty)$

f) Interval of decrease: $(-\infty, 2)$

3) (6 points) Inspired by the game *Stardew Valley Crossing*, Mike decides to take up farming. He plans to build 4 adjacent, rectangular pens enclosed on all sides. He has 400 feet of fencing available. Determine a function that will relate the area of the enclosure to the width x and algebraically find the width x of the enclosure that maximizes the area. Also, what is the maximum area?



$5x + 2y = 400$

$y = 200 - 5/2\,x$

$A = xy$

$= x(200 - 5/2\,x)$

$A(x) = -5/2\,x^2 + 200x$

$x = -\frac{b}{2a} = \frac{-200}{2(-5/2)} = 40$

$A(40) = 4000$

4) (4 points) Solve for the variable. $\frac{3x-7}{x^2-9} - \frac{5}{x-3} = \frac{7}{x+3}$ $\quad (x+3)(x-3)$

$3x - 7 - 5(x+3) = 7(x-3)$

$3x - 7 - 5x - 15 = 7x - 21$

$-9x = 1$

$x = -1/9$

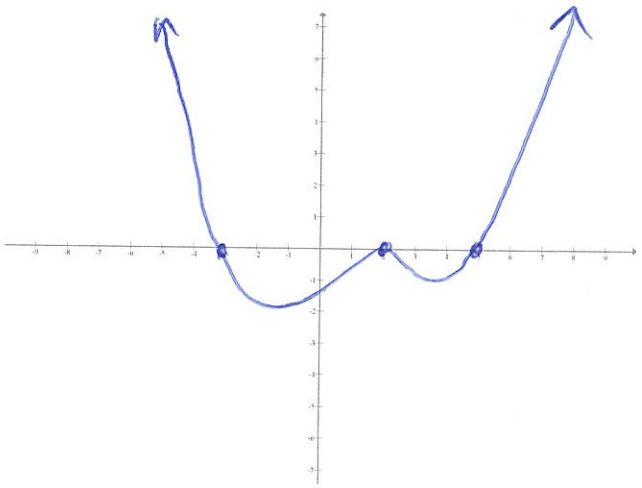
5) (3 points each) For the function $f(x) = (x+3)(x-2)^2(x-5)\ldots$

a) Find the leading term and state which quadrants the arrowheads will be in and why:

LT: x^4 → even
↳ positive

Q I & II

c) Sketch the graph based on parts *a* and *b*:



b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
-3	1	cross
2	2	touch
5	1	cross

$-3x^2(x^2-4) = -3x^2(x+2)(x-2)$

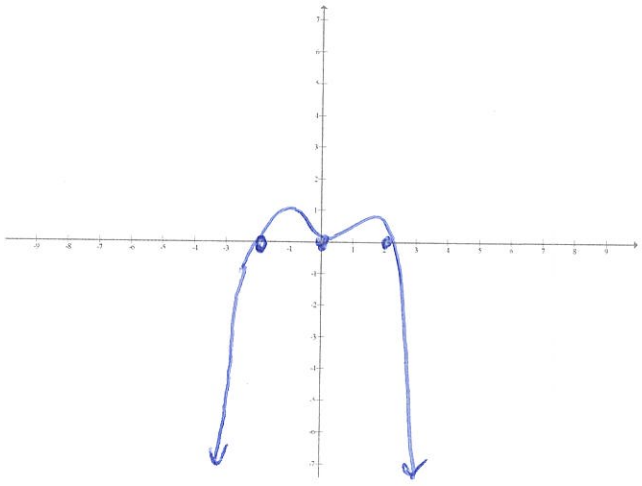
6) (3 points each) For the function $f(x) = -3x^4 + 12x^2\ldots$

a) Find the leading term and state which quadrants the arrowheads will be in and why:

LT: $-3x^4$ → even
↳ neg

Q III, IV

c) Sketch the graph based on parts *a* and *b*:



b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
0	2	touch
-2	1	cross
2	1	cross

7) (1 point each) Fill in the blank:

If c is a zero of a function f, then $f(c) =$ _____ _ _____, and _____ is a factor.

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8) (2 points each) Form a polynomial <u>function</u> of degree four that meets the following requirements. **Be sure to leave your answer in factored form**:

a) Has zeros at 2, 1, 6, and −4:

$f(x) = (x-2)(x-1)(x-6)(x+4)$

b) Has the same zeros and multiplicity as in part *a* but is a different function:

$g(x) = (12)(x-2)(x-1)(x-6)(x+4)$

(any non-zero number)

c) Has a zero at $1 - 5i$, and 8 is a zero of multiplicity 2:

$h(x) = (x-(1-5i))(x-(1+5i))(x-8)^2$

9) (3 pts *a;* 2 pts others) Consider the functions $f(x) = 6x^3 + x^2 - 12x + 5$ and $g(x) = 3x^2 + 2x + 1$.

a) Divide $f(x)$ by $g(x)$ using long division:

$$3x^2+2x+1 \overline{)\,6x^3+x^2-12x+5}$$

$2x - 1$

$-(6x^3+4x^2+2x)$

$-3x^2-14x+5$

$-(-3x^2-2x-1)$

$-12x+6$

b) Based on your work in part *a*, was $g(x)$ a factor of $f(x)$? Why or why not?

Nope, remainder wasn't zero.

c) What is the equation of the oblique asymptote of the rational function $y = \frac{6x^3+x^2-12x+5}{3x^2+2x+1}$?

$y = 2x - 1$

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10) (8 points) Factor the polynomial completely by first listing the possible rational roots and then using synthetic division and your calculator.

$$g(x) = x^4 - 6x^3 + 22x^2 - 48x + 40$$

$p = \pm 1, 2, 4, 5, 8, 10, 20, 40$

$q = \pm 1$

$\frac{p}{q} = \pm 1, 2, 4, 5, 8, 10, 20, 40$

2	1	−6	22	−48	40
	↓	2	−8	28	−40
2	1	−4	14	−20	0
	↓	2	−4	20	
	1	−2	10	0	

$x^2 - 2x + 10 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-36}}{2} = 1 \pm 3i$

$g(x) = (x-2)^2(x-(1+3i))(x-(1-3i))$

11) (3 points each) For the function $f(x) = \frac{x+1}{x^2-2x-3}$, find…

a) The domain:

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x \neq 3, -1$

b) The x- and y-intercepts:

x-int: $x + 1 = 0$ $x = -1$ → $\emptyset$

y-int: $f(0) = -\frac{1}{3}$ $\left(0, -\frac{1}{3}\right)$

c) Any vertical asymptotes and holes:

$x = 3$: $3 + 1 \neq 0$ ✓ $x = 3$ VA

$x = -1$: $-1 + 1 = 0$ ✓ H.O.

$\frac{x+1}{(x-3)(x+1)} = \frac{1}{x-3}$

@ $x = -1$: $\frac{1}{-1-3} = -\frac{1}{4}$

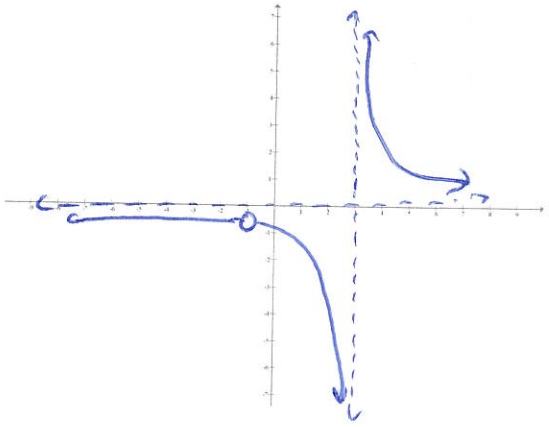
H.O. $(-1, -1/4)$

d) Any horizontal or oblique asymptotes:

$y = 0$ HA

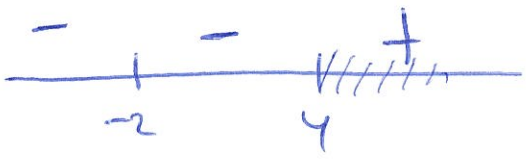
e) Sketch a graph based on the work above.

Hint: you should see a transformation of $y = \frac{1}{x}$ *in your work. Use that!*



12) (3 points) Solve for the variable. Write answer in interval notation:

$$(x-4)(x+2)^2 > 0$$



$(4, \infty)$

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