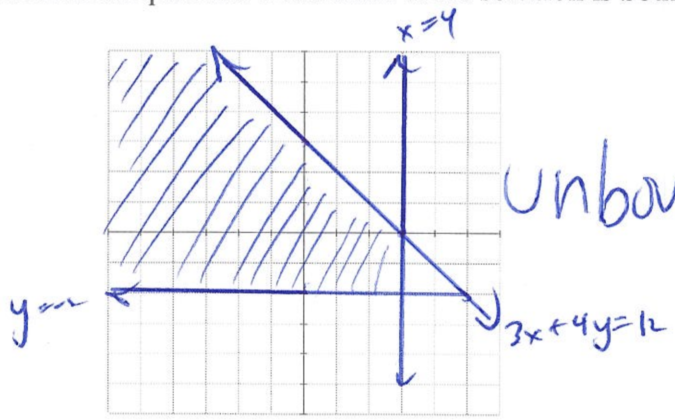
1) (6 points) Sketch the following system of linear inequalities. **Determine if the solution is bounded or unbounded.**

$$\begin{cases} 3x+4y\le 12 \\ x\le 4 \\ y\ge -2 \end{cases}$$




Unbounded

2) Solve the following Maximization LP using the corner point method:

In order to maximize viewers, the MNT network plans to broadcast a 12-hour marathon of two shows: *Gerbils of Doom* and *Let's Make a Donut*. The show *Gerbils* attracts 2.2 million viewers and runs for 60 minutes while *Donut* attracts 1.4 million viewers and runs for 30 minutes. Due to contract with both shows, they each need to appear at least 4 times during the marathon. How many times should each show be run to maximize the total number of viewers during the marathon? What is the maximum number of viewers during the marathon?

a) (8 points) Name and define variables. Write an objective function and all necessary constraints:

Maximize $z=2.2x+1.4y$

subject to

hours $1x+\frac{1}{2}y\le 12$

contract $x\ge 4$

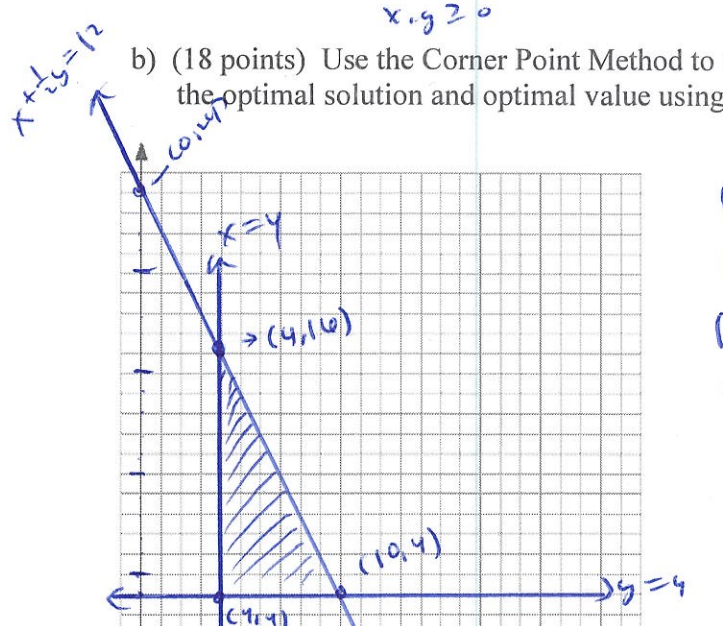
$y\ge 4$

$x,y\ge 0$

$x$ = # of runs of GoD

$y$ = # of runs of LMaD

b) (18 points) Use the Corner Point Method to solve the LP. **Count the tick-marks by 2.** Interpret the optimal solution and optimal value using the language of the problem.



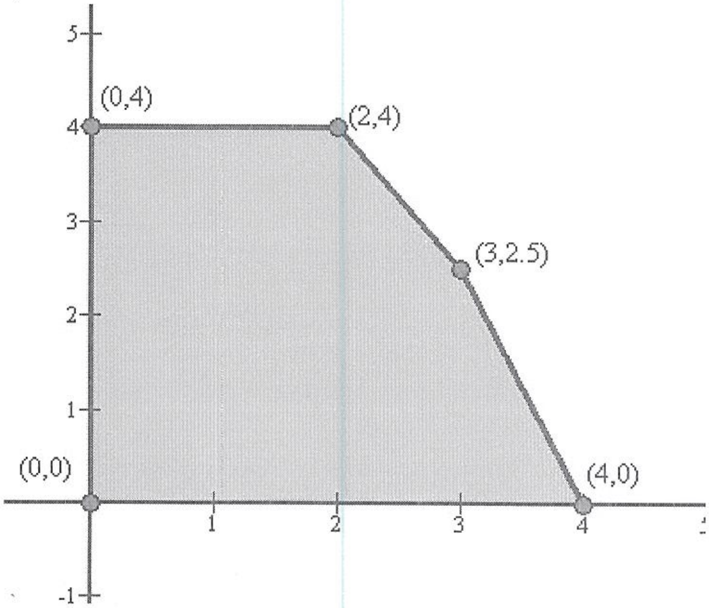

| (x,y) | z=2.2x+1.4y |
|---|---|
| (4,4) | 14.4 |
| (10,4) | 27.6 |
| (4,16) | 31.2 |

They should run GoD 4 times and LMaD 16 times for a max # of 31.2 million viewers.

32

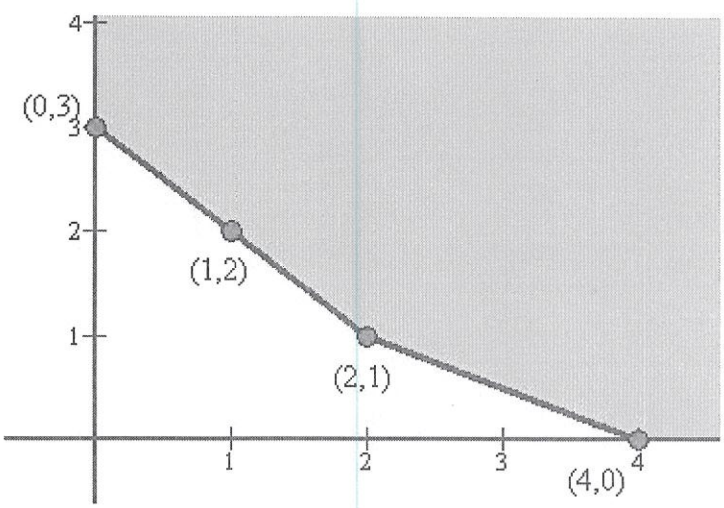
3) (5 points each) For the following feasible regions, find the maximum and minimum of the objective function $z = 5x + 3y$ if they exist. **Be sure to label your answers as "maximum" and minimum".**

a)




| $(x, y)$ | $z = 5x + 3y$ |
|---|---|
| (0, 0) | 0 min |
| (0,4) | 12 |
| (2,4) | 22 |
| (3, 2.5) | 22.5 MAX |
| (4,0) | 20 |

b)




| $(x, y)$ | $z = 5x + 3y$ |
|---|---|
| (0,3) | 9 min |
| (1,2) | 11 |
| (2,1) | 13 |
| (4,0) | 20 |

No MAX

10

4) (3 points each) Consider a Maximization LP in standard form…

a) How is Pivot Column determined?

do

b) Based on your answer from part *a*, why is the Pivot Column picked this way?

bee

c) Graphically, what does the Pivot Column tell you?

do

d) How is the Pivot Row determined?

bee

e) Based on your answer to part *d*, why is the Pivot Row picked this way?

f) Graphically, what does the Pivot Row tell you?

dooooooo

5) (6 points each) The following tableaus need pivoting. Circle the pivot element or explain why there isn't one. **Do not actually pivot.** Also state what the current value of *z* is and the corresponding augmented coordinates:

a)

| $BV$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $z$ | $RHS$ |
|---|---|---|---|---|---|---|
| $s_1$ | 1 | 3 | 1 | 0 | 0 | 550 |
| $s_2$ | 4 | 2 | 0 | 1 | 0 | 220 |
| $z$ | −1 | −2 | 0 | 0 | 1 | 0 |

z = 0

(0, 0, 550, 220)

b)

| $BV$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $z$ | $RHS$ |
|---|---|---|---|---|---|---|---|
| $s_1$ | 1 | −6 | 0 | 1 | 1 | 0 | 600 |
| $x_3$ | 3 | 0 | 1 | 0 | 2 | 0 | 250 |
| $z$ | 1 | −5 | 0 | 0 | 8 | 1 | 0 |

No pivot element → element in pivot column are not positive

z = 0

(0, 0, 250, 600, 0)

30

6) For the following Minimization LP…

Minimize $w = 16y_1 + 11y_2 + 15y_3$

subject to

$$2y_1 + y_2 + y_3 \geq 3$$
$$y_1 + 2y_2 + 3y_3 \geq 5$$
$$y_1, y_2, y_3 \geq 0$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 1 & 2 & 3 & 5 \\ \hline 16 & 11 & 15 & 0 \end{array}\right]^T = \left[\begin{array}{cc|c} 2 & 1 & 16 \\ 1 & 2 & 11 \\ 1 & 3 & 15 \\ \hline 3 & 5 & 0 \end{array}\right]$$

a) (8 points) Write the dual Maximization LP:

MAXimize $z = 3x_1 + 5x_2$

subject to

$$2x_1 + x_2 \leq 16$$
$$x_1 + 2x_2 \leq 11$$
$$x_1 + 3x_2 \leq 15$$
$$x_1, x_2 \geq 0$$

b) (12 points) Use the Simplex Method on the LP found from part *a*:

| BV | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $z$ | RHS |
|---|---|---|---|---|---|---|---|
| $s_1$ | 2 | 1 | 1 | 0 | 0 | 0 | 16 |
| $s_2$ | 1 | 2 | 0 | 1 | 0 | 0 | 11 |
| $s_3$ | 1 | 3 | 0 | 0 | 1 | 0 | 15 |
| $z$ | -3 | -5 | 0 | 0 | 0 | 1 | 0 |

$\frac{1}{3}R_3 \to R_3$
$-R_3 + R_1 \to R_1$
$-2R_3 + R_2 \to R_2$
$5R_3 + R_4 \to R_4$

| BV | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $z$ | RHS |
|---|---|---|---|---|---|---|---|
| $s_1$ | 5/3 | 0 | 1 | 0 | -1/3 | 0 | 11 |
| $s_2$ | 1/3 | 0 | 0 | 1 | -2/3 | 0 | 1 |
| $x_2$ | 1/3 | 1 | 0 | 0 | 1/3 | 0 | 5 |
| $z$ | -4/3 | 0 | 0 | 0 | 5/3 | 1 | 25 |

$3R_2 \to R_2$
$-\frac{5}{3}R_2 + R_1 \to R_1$
$-\frac{1}{3}R_2 + R_3 \to R_3$
$\frac{4}{3}R_2 + R_4 \to R_4$

| BV | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $z$ | RHS |
|---|---|---|---|---|---|---|---|
| $s_1$ | 0 | 0 | 1 | -5 | 3 | 0 | 6 |
| $x_1$ | 1 | 0 | 0 | 3 | -2 | 0 | 3 |
| $x_2$ | 0 | 1 | 0 | -1 | 1 | 0 | 4 |
| $z$ | 0 | 0 | 0 | 4 | -1 | 1 | 29 |

$\frac{1}{3}R_1 \to R_1$
$2R_1 + R_2 \to R_2$
$-R_1 + R_3 \to R_3$
$R_1 + R_4 \to R_4$

| BV | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $z$ | RHS |
|---|---|---|---|---|---|---|---|
| $s_3$ | 0 | 0 | 1/3 | -5/3 | 1 | 0 | 2 |
| $x_1$ | 1 | 0 | 2/3 | -1/3 | 0 | 0 | 7 |
| $x_2$ | 0 | 1 | -1/3 | 2/3 | 0 | 0 | 2 |
| $z$ | 0 | 0 | 1/3 | 7/3 | 0 | 1 | 31 |

c) (8 points) What is the optimal solution and optimal value of the Minimization LP?

$w = 31 \quad y_1 = 1/3 \quad y_2 = 7/3 \quad y_3 = 0$

Extra Credit: What is the optimal solution and optimal value of the Maximization LP?

$z = 31 \quad x_1 = 7 \quad x_2 = 2$

28/30