1) (4 points each) Simplify:

a) $\tan x(\tan x+\cot x)$

$= \tan^2 x + 1$

$= \boxed{\sec^2 x}$

b) $\dfrac{\tan x}{\tan x+\cot x}$

$= \dfrac{\frac{\sin x}{\cos x}\cdot \sin x\cos x}{\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)\cdot \sin x\cos x} = \dfrac{\sin^2 x}{\sin^2 x+\cos^2 x}$

$= \dfrac{\sin^2 x}{1} = \boxed{\sin^2 x}$

2) (4 points each) Find the exact value of $\tan\dfrac{\pi}{12}$ using the given methods. Clean up any complex fractions, but you do not have to rationalize your final answer.

a) A Sum or Difference Formula:

$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\cdot\tan\frac{\pi}{4}}$

$= \boxed{\dfrac{\sqrt{3}-1}{1+\sqrt{3}}}$

b) A Half Angle Formula:

$\tan\frac{\pi}{12} = \tan\dfrac{\pi/6}{2} = \dfrac{1-\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$

$= \dfrac{\left(1-\frac{\sqrt{3}}{2}\right)\cdot 2}{\left(\frac{1}{2}\right)\cdot 2} = \boxed{2-\sqrt{3}}$

c) Using your answer from either part *a* or *b* above, **explain** how you can find the exact value of $\tan\left(\dfrac{23\pi}{12}\right)$ by using $\dfrac{\pi}{12}$ as a reference angle.

because science

3) (5 points each) Simplify:

a) $\dfrac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}$

$= \dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta-(\sin\alpha\cos\beta-\cos\alpha\sin\beta)}{\cos\alpha\cos\beta-\sin\alpha\sin\beta-(\cos\alpha\cos\beta+\sin\alpha\sin\beta)}$

$= \dfrac{\cancel{2}\cos\alpha\,\cancel{\sin\beta}}{-\cancel{2}\sin\alpha\,\cancel{\sin\beta}} = \boxed{-\cot\alpha}$

b) $\dfrac{\sin^2\alpha}{\tan^2\alpha}-\dfrac{\tan^2\alpha}{\sec^2\alpha}$

$= \dfrac{\sin^2\alpha}{\frac{\sin^2\alpha}{\cos^2\alpha}} - \dfrac{\frac{\sin^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}}$ flip & multiply

$= \cos^2\alpha - \sin^2\alpha$

$= \boxed{\cos 2\alpha}$

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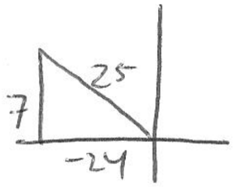


4) (5 points each) Given $\sin\theta = \dfrac{7}{25}$, where θ is in Quadrant II, find the exact values for…

a) $\sin(2\theta)$

$= 2\left(\frac{7}{25}\right)\left(-\frac{24}{25}\right) = \boxed{-\frac{336}{625}}$

b) $\cos(2\theta)$

$= \left(-\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2$

$= \boxed{\frac{527}{625}}$

c) $\tan(2\theta)$

$= \boxed{-\frac{336}{527}}$

d) The Quadrant where 2θ resides. Explain why:

QIV

$\cos 2\theta > 0$
$\sin 2\theta < 0$
$\tan 2\theta$

5) (4 points each) Find the exact values or explain why it does not exist:

a) $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{5\pi}{6}}$

b) $\cos\left(\cos^{-1}(-1.1)\right)$

DNE $-1.1 \notin D_{\cos^{-1}}$

c) $\cos^{-1}\left(\cos\left(-\dfrac{2\pi}{3}\right)\right)$

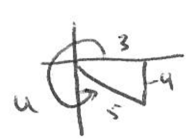
$= \cos^{-1}\left(-\frac{1}{2}\right)$

$= \frac{2\pi}{3}$

d) $\sin\Big[\underbrace{\sin^{-1}\left(-\dfrac{4}{5}\right)}_{u} - \underbrace{\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)}_{v}\Big]$

$= \sin\left(\sin^{-1}\left(-\frac{4}{5}\right)\right)\cos\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) -$
$\cos\left(\sin^{-1}\left(-\frac{4}{5}\right)\right)\sin\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= \left(-\frac{4}{5}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{3}{5}\right)\left(\frac{1}{2}\right) = \boxed{\frac{4\sqrt{3}-3}{10}}$



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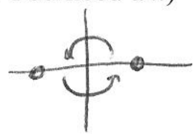
6 ~~7~~) (6 points each) Solve for the variable:

a) $\cos^2 x - 1 = 0$ (Hint: You need a k)

$\cos^2 x = 1$

$\cos x = \pm 1$



$\boxed{x = 0 + k \cdot \pi, k \in \mathbb{Z}}$

b) $\sin(2x) = \frac{\sqrt{3}}{2}$ on $[0, 2\pi)$

$2x = \frac{\pi}{3} + 2\pi k \Rightarrow x = \frac{\pi}{6} + \pi k$

$2x = \frac{2\pi}{3} + 2\pi k \Rightarrow x = \frac{\pi}{3} + \pi k$

$\left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}\right\}$

7 ~~8~~) (5 points) Fill in the blank using interval notation:

	$\sin x$ *	$\cos x$ *	$\tan x$ *	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain						
Range						

Free Boxes!!!

*Write the domain restrictions for these three functions.

8 ~~9~~) (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

cuz

Extra Credit:

Simplify: $\cos\left(\frac{\pi}{3} - \alpha\right)\cos\left(\frac{\pi}{3} + \alpha\right) - \sin\left(\frac{\pi}{3} - \alpha\right)\sin\left(\frac{\pi}{3} + \alpha\right)$

$-\frac{1}{2}$

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