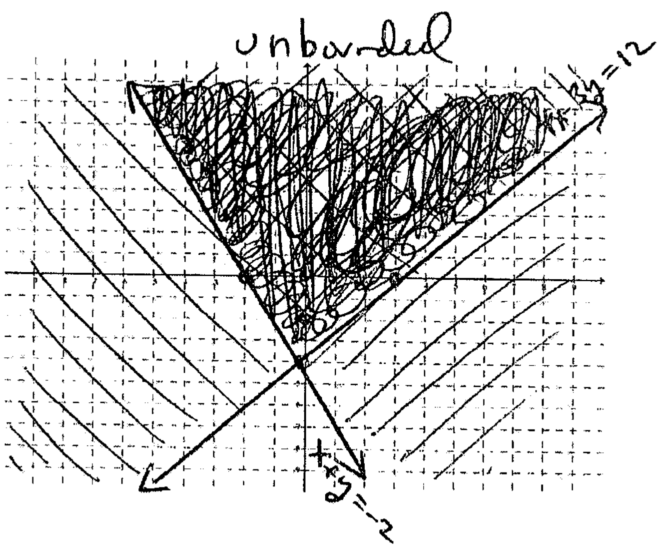
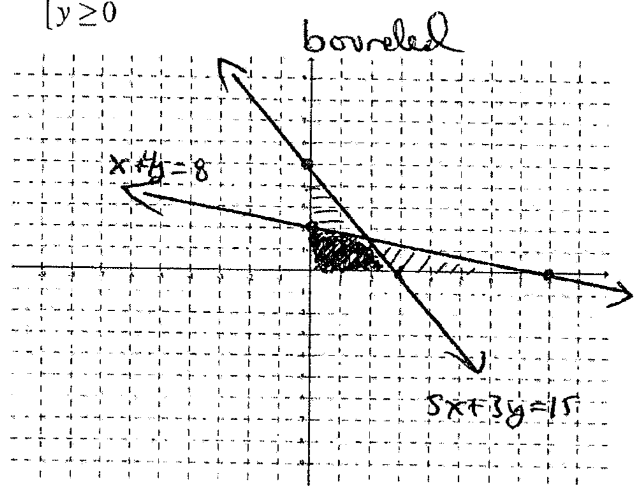
1) (6 points each) For the following systems of equations, sketch the feasible region and determine if the feasible region is bounded or unbounded. Be sure to shade your answer in the darkest:

a) $\begin{cases} 4x-3y \leq 12 \\ x+y \geq -2 \end{cases}$



b) $\begin{cases} x+4y \leq 8 \\ 5x+3y \leq 15 \\ x \geq 0 \\ y \geq 0 \end{cases}$



2) (14 points) For the following linear programming problem:

i) Graph the feasible region and label the corner points. **Use fractions as needed.**

ii) Use the Corner Point Method to solve. Label the optimal solution and optimal value.

Minimize $z = 10x + 42y$

subject to

$x + 7y \geq 6$

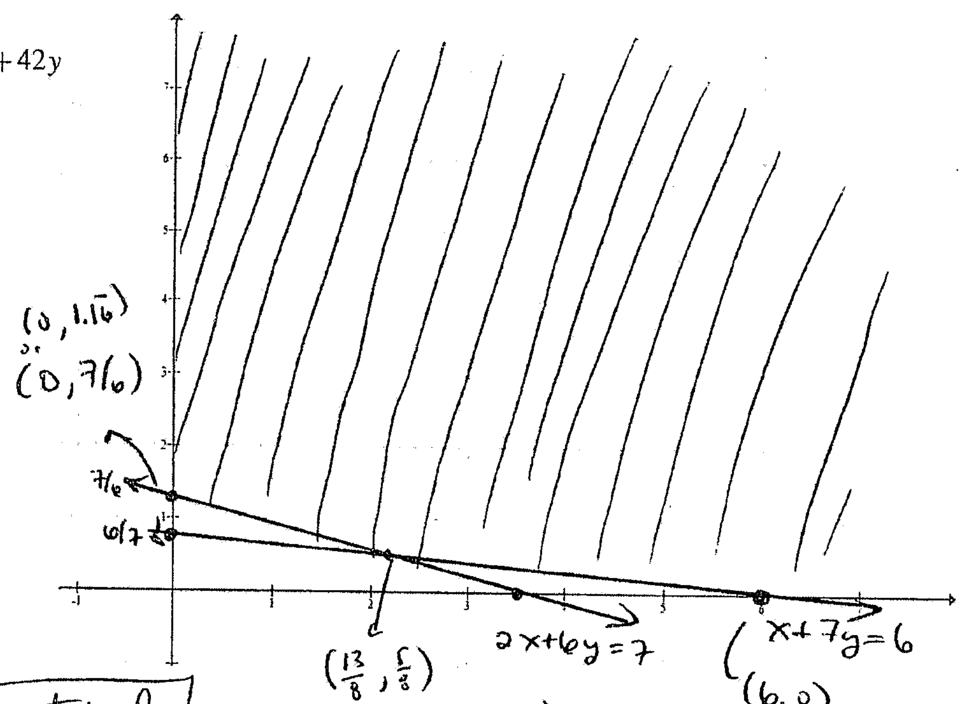
$2x + 6y \geq 7$

$x \geq 0$

$y \geq 0$

(x,y)	$z = 10x + 42y$
$(0, \frac{7}{6})$	49
$(\frac{13}{8}, \frac{5}{8})$	42.5
$(6, 0)$	60

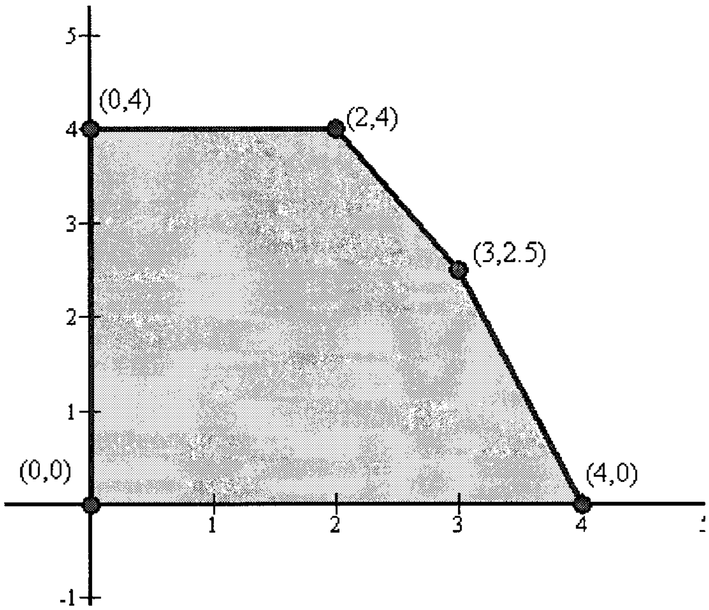
optimal sol'n	optimal value
$(\frac{13}{8}, \frac{5}{8})$	42.5



26

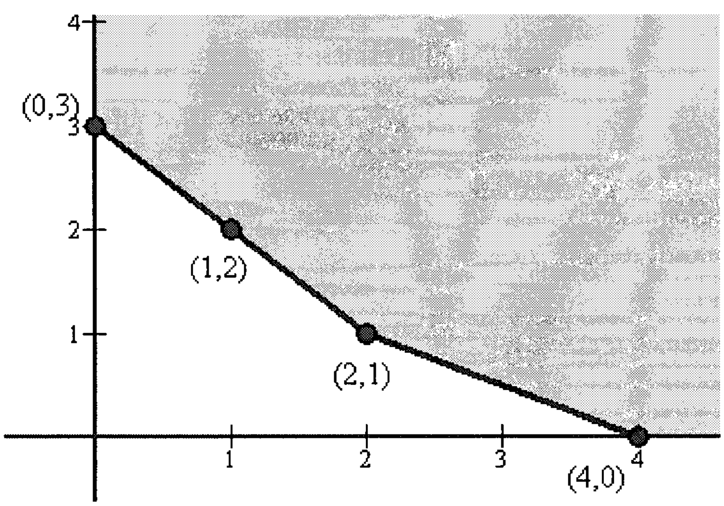
3) (6 points each) For the following feasible regions, find the maximum and minimum of the objective function $z = 2x + 7y$ if they exist. **Be sure to label your answers as "maximum" and minimum".**

a)



(x, y)	$z = 2x + 7y$
(0, 0)	0 ← min
(0, 4)	28
(2,4)	32 ← max
(3, 2.5)	23.5
(4, 0)	8

b)



(x, y)	$z = 2x + 7y$
(0,3)	21
(1,2)	16
(2,1)	11
(4,0)	8 ← min
	No MAX!

12

4) For the following linear programming problem:

Dandy Candy makes and sells many types of candy bars. Their top three sellers are the Peanut Amazement, the Crunchy Yum, and the Sweet Tooth. Each Peanut Amazement needs 40 peanuts, ¼ cup of caramel, 1.5 hours to bake and sells for $3.50. Each Crunchy Yum needs 60 peanuts, ½ cup of caramel, 2 hours to bake and sells for $4.75. Each Sweet Tooth needs 15 peanuts, 2 cups of caramel, 1 hour to bake and sells for $2.75. If there are at most 500 peanuts, 20 cups of caramel, and 45 hours of baking time available, how many of each type of candy bar should be sold to maximize revenue?

a) (3 points) Name and define variables for this problem:

x_1 = # of Peanut Amazement

x_2 = # of Crunchy Yum

x_3 = # of Sweet Tooth

b) (6 points) Write the corresponding linear programming problem including all inequalities:

Maximize $z = 3.50x_1 + 4.75x_2 + 2.75x_3$

subject to

$$40x_1 + 60x_2 + 15x_3 \le 500$$
$$\tfrac{1}{4}x_1 + \tfrac{1}{2}x_2 + 2x_3 \le 20$$
$$1.5x_1 + 2x_2 + 1x_3 \le 45$$
$$x_1, x_2, x_3 \ge 0$$

c) (4 points) Write the linear programming problem in equation form by introducing slack variables. **You do not have to define your slack variables.**

Maximize $z = 3.50x_1 + 4.75x_2 + 2.75x_3$

subject to

$$40x_1 + 60x_2 + 15x_3 + s_1 = 500$$
$$\tfrac{1}{4}x_1 + \tfrac{1}{2}x_2 + 2x_3 + s_2 = 20$$
$$1.5x_1 + 2x_2 + 1x_3 + s_3 = 45$$
$$x_1, x_2, x_3 \ge 0$$

d) (4 points) Write the initial simplex tableau for this problem. **Do not solve.**

BV	x_1	x_2	x_3	s_1	s_2	s_3	z	RHS
s_1	40	60	15	1	0	0	0	500
s_2	$\frac{1}{4}$	$\frac{1}{2}$	2	0	1	0	0	20
s_3	1.5	2	1	0	0	1	0	45
z	−3.50	−4.75	−2.75	0	0	0	1	0

17

5) (2 points each) Consider a Maximization LP in standard form…

a) How is Pivot Column determined?

b) Based on your answer from part *a*, why is the Pivot Column picked this way?

c) What does the Pivot Column tell us geometrically?

d) How is the Pivot Row determined?

e) Based on your answer to part *d*, why is the Pivot Row picked this way?

f) What does the Pivot Row tell us geometrically?

6) (9 points) For the tableau below:

i) Fill in the Basic Variable column.

ii) Write the corresponding point (x_1, x_2, s_1, s_2) and objective function value. Is it a corner point? Why or why not?

iii) Circle the pivot element.

iv) Declare what the new Basic Variable and Non-basic Variable will be *after* the pivot. **Do not actually pivot**

new BV

BV	x_1	x_2	s_1	s_2	z	RHS	
x_1	1	4	0	6	0	8	$8/4 = 2$
s_1	0	6	1	5	0	10	$\frac{10}{6} = 1.\overline{6}$
z	0	-12	0	-6	1	27	

new non BV

Point corresponding to tableau: (8, 0, 10, 0)

Yes cp since 2 zero's & the rest are positive

Obj. function value: 27

21

7) For the following Minimization LP...

$$\text{Minimize } w = 10y_1 + 42y_2$$
subject to
$$y_1 + 7y_2 \geq 6$$
$$2y_1 + 6y_2 \geq 7$$
$$y_1 \geq 0$$
$$y_2 \geq 0$$

I've seen this somewhere...

a) (6 points) Write the dual Maximization LP. Be sure to use the correct variables!

$$\left[\begin{array}{cc|c} 1 & 7 & 6 \\ 2 & 6 & 7 \\ \hline 10 & 42 & 0 \end{array}\right]^T = \left[\begin{array}{cc|c} 1 & 2 & 10 \\ 7 & 6 & 42 \\ \hline 6 & 7 & 0 \end{array}\right]$$

Maximize $z = 6x_1 + 7x_2$
subject to
$x_1 + 2x_2 \leq 10$
$7x_1 + 6x_2 \leq 42$
$x_1, x_2 \geq 0$

b) (15 points) Use the Simplex Method to solve the Max LP found from part *a*. Show all necessary work:

BV	x_1	x_2	s_1	s_2	z	RHS
s_1	1	2	1	0	0	10
s_2	7	6	0	1	0	42
z	-6	-7	0	0	1	0

$\frac{1}{2}R_1 \to R_1$
$-6R_1 + R_2 \to R_2$
$7R_1 + R_3 \to R_3$

BV	x_1	x_2	s_1	s_2	z	RHS
x_2	1/2	1	1/2	0	0	5
s_2	4	0	-3	1	0	12
z	-5/2	0	7/2	0	1	35

$\frac{1}{4}R_2 \to R_2$
$-\frac{1}{2}R_2 + R_1 \to R_1$
$\frac{5}{2}R_2 + R_3 \to R_3$

BV	x_1	x_2	s_1	s_2	z	RHS
$\cancel{x}_2$	0	1	7/8	-1/8	0	7/2
x_1	1	0	-3/4	1/4	0	3
z	0	0	13/8	5/8	1	85/2

c) (3 points) What is the optimal solution and optimal value of the Minimization LP?

$$\boxed{(13/8, 5/8) \quad w = 85/2 = 42.5}$$
$y_1 \quad y_2$

34