1) (12 points each) Solve the system using the methods listed below. Write answer as an ordered triple.

$$\begin{cases} 2x+y+2z=2 \\ 4x-7y+8z=-10 \\ x-2y-z=-6 \end{cases}$$

a) The Elimination method:

① $2x+y+2z=2$
$-2x+4y+2z=12$
$5y+4z=14$

② $-4x-2y-4z=-4$
$4x-7y+8z=-10$
$-9y+4z=-14$

③ $-5y-4z=-14$
$-9y+4z=-14$
$-14y=-28$

④ $y=2$

⑤ $5(2)+4z=14$
$z=1$

$x-2(2)-(1)=-6$
$x=-1$ $(-1,2,1)$

b) Gauss-Jordan method:

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 2 \\ 4 & -7 & 8 & -10 \\ 1 & -2 & -1 & -6 \end{array}\right] \xrightarrow[\substack{-4R_1+R_2\to R_2 \\ -2R_1+R_3\to R_3}]{R_1\leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -6 \\ 0 & 1 & 12 & 14 \\ 0 & 5 & 4 & 14 \end{array}\right] \xrightarrow[-5R_2+R_3\to R_3]{2R_2+R_1\to R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 23 & 22 \\ 0 & 1 & 12 & 14 \\ 0 & 0 & -56 & -56 \end{array}\right] \xrightarrow[\substack{-23R_3+R_1\to R_1 \\ -12R_3+R_2\to R_2}]{-\frac{1}{56}R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad (-1,2,1)$$

2) (1 point) Verify that you made absolutely sure that your answer to 1a is the same as in 1b by signing your name here Jake Peralta. You will not receive the credit if the work does not support the same answer.

3) (4 points each) The following systems are special cases. Mark your answer as either "no solution" or "infinitely many solutions" and show supporting work. You may use Elimination or Gauss-Jordan to determine the special case.

a) $\begin{cases} 5x-3y=15 \\ -10x+6y=8 \end{cases}$ (2)

$10x-6y=30$
$-10x+6y=8$
$0=32$

$\boxed{\text{No solution}}$

b) $\begin{cases} x+2y+z=4 \\ 3x+4y-z=8 \\ 4x+6y \quad =12 \end{cases}$

$4x+6y=12$
$\to -4x-6y=-12$
$0=0$

$\boxed{\text{infinitely many sol'n}}$

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4) For the following problem:

A person invested \$4,200 for one year, part at 8%, part at 10%, and the remainder at 12%. The total annual return was \$716. The total amount of money invested in the 12% was \$300 more than the amounts invested at 8% and 10% combined. How much was invested at each rate?

a) (3 points) Name and define your variables for this problem:

x = amt invested in 8%
y = " " " 10%
z = " " " 12%

b) (5 points) Set up **BUT DO NOT SOLVE** a system of equations for this problem:

$$\begin{cases} x + y + z = 4200 \\ 0.08x + 0.10y + 0.12z = 716 \\ z = 300 + x + y \end{cases} \xrightarrow{\text{or}} (-x - y + z = 300)$$

5) (2 points each) For the matrix: $A = \begin{bmatrix} 5 & 7 & 10 & -2 & 0 \\ 1 & 4 & -8 & 6 & \frac{1}{2} \end{bmatrix}$, determine...

a) The dimension of matrix A

2×5

b) The 2,3 entry

-8

6) (4 points each) For the following matrices:

$A = \begin{bmatrix} 5 & -2 \\ 9 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 6 & -2 & 4 \\ 1 & 12 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 6 & 1 \\ -4 & -2 \end{bmatrix}$ $D = \begin{bmatrix} -2 & 8 \\ 5 & 12 \\ 3 & 4 \end{bmatrix}$

Find the following or explain why they do not exist:

a) $4A - 5C$

$\begin{bmatrix} -10 & -13 \\ 56 & 10 \end{bmatrix}$

b) $6B + D$

DNE
Not the same dimension

c) BD

$= \begin{bmatrix} -10 & 40 \\ 61 & 156 \end{bmatrix}$

24

7) (2 points each) What property must be true to…

a) Add or subtract matrices?

b) Multiply matrices?

must love dogs

see part a

8) (6 points each) From Monday – Friday, you like to spend 2 hours playing video games, 1 hour reading, and 3 hours doing math homework each day. On Saturday and Sunday, you like to spend 4 hours playing video games, no time reading, and 6 hours doing math homework each day.

a) Create two 1×3 matrices, called M and S respectively, where the first shows the amount of time spent on the three activities on Monday **only** and the other shows the amount of time spent on the three activities on Saturday **only**. Be sure to label the rows and columns.

$$M = \begin{array}{cc} & \begin{array}{ccc} vg & read & mh \end{array} \\ hr & \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \end{array} \qquad S = \begin{array}{cc} & \begin{array}{ccc} vg & read & mh \end{array} \\ hr & \begin{bmatrix} 4 & 0 & 6 \end{bmatrix} \end{array}$$

b) Using scalar multiplication and addition, find the total time spent on the three activities for the entire week. Be sure to label the rows and columns.

$$5M + 2S = \begin{array}{cc} & \begin{array}{ccc} vg & read & mh \end{array} \\ hr & \begin{bmatrix} 18 & 5 & 27 \end{bmatrix} \end{array}$$

9) (6 points each) Brothers Romulus and Remus do chores at home to earn an allowance. They plan to establish the city of Rome with all of their earnings. Romulus will do yardwork 2 times a week, wash the dishes 3 times a week, walk the dog 4 times a week, and vacuum 1 time a week. Remus will do yardwork 1 time a week, wash the dishes 4 times a week, walk the dog 2 times a week, and vacuum 3 time a week. They are paid \$5 every time they do yardwork, \$2 every time they wash the dishes, \$7 every time they walk the dog, and \$4 every time they vacuum.

a) Create a 2×4 matrix called B showing the names of the brothers and the number of times they do the given chores. Create a 4×1 matrix called A showing the amount paid for each chore. Be sure to label the rows and columns.

$$B = \begin{array}{cc} & \begin{array}{cccc} yard & dish & dog & vacuum \end{array} \\ \begin{array}{c} Romulus \\ Remus \end{array} & \begin{bmatrix} 2 & 3 & 4 & 1 \\ 1 & 4 & 2 & 3 \end{bmatrix} \end{array} \qquad A = \begin{array}{cc} & \$ \\ \begin{array}{c} yard \\ dish \\ dog \\ vacuum \end{array} & \begin{bmatrix} 5 \\ 2 \\ 7 \\ 4 \end{bmatrix} \end{array}$$

b) Find the product BA and interpret each value. Be sure to label the rows and columns.

$$BA = \begin{array}{cc} & \$ \\ \begin{array}{c} Romulus \\ Remus \end{array} & \begin{bmatrix} 48 \\ 39 \end{bmatrix} \end{array}$$

money earned per week for each brother

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10) (12 points part *a*; 3 points part *b*) For the system $\begin{cases} 3x - y = 2 \\ x - 2y + 2z = -2 \\ 2x - 3y + 3z = -1 \end{cases}$ ...

a) Find the inverse of the coefficient matrix algebraically using the Gauss-Jordan Method:

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 0 & 1 & 0 & 0 \\ 1 & -2 & 2 & 0 & 1 & 0 \\ 2 & -3 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\substack{-3R_1+R_2 \to R_2 \\ -2R_1+R_3 \to R_3}]{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -6 & 1 & -3 & 0 \\ 0 & 1 & -1 & 0 & -2 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{2R_2+R_1 \to R_1 \\ -5R_2+R_3 \to R_3}]{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & -1 & 0 & -2 & 1 \\ 0 & 0 & -1 & 1 & 7 & -5 \end{array}\right] \xrightarrow[R_3+R_2 \to R_2]{-R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & -1 & -9 & 6 \\ 0 & 0 & 1 & -1 & -7 & 5 \end{array}\right]$$

answer

b) Solve the system using the matrix inverse from part *a*. Write answer as an ordered triple.

$$\begin{bmatrix} 0 & -3 & 2 \\ -1 & -9 & 6 \\ -1 & -7 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{array}{l} \leftarrow x \quad 4 \\ \leftarrow y \quad 10 \\ \leftarrow z \quad 7 \end{array}$$

(   )

4   10   7

TS