1) (5 points) Solve for the triangle. Be sure to show all necessary work:

$A = 42°$ $\quad a = 15$ $\quad \frac{b}{\sin 57°} = \frac{15}{\sin 42°} \Rightarrow b = \frac{15 \sin 57°}{\sin 42°} \approx 19$

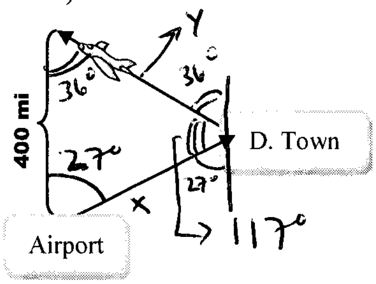
$B = 57°$ $\quad b = \underline{19}$ $\quad \frac{c}{\sin 81°} = \frac{15}{\sin 42°} \Rightarrow c = \frac{15 \sin 81°}{\sin 42°} \approx 22$

$C = \underline{81}°$ $\quad c = \underline{22}$

$180° - (42° + 57°)$

2) (6 points) Math Airways drops calculators to students in need. A plane leaves the airport bearing $N27°E$ towards Division Town. While flying over Division Town, the plane heads on a bearing of $N36°W$. After some time, it is 400 miles due north of the airport. See picture:

a) Find the total number of miles the plane flew to and from Division Town. Round only at the end:



$$X = \frac{400 \sin 36°}{\sin 117°} \Rightarrow X \approx 264 \text{ mi}$$

$$y = \frac{400 \sin 27°}{\sin 117°} \Rightarrow y \approx 204 \text{ mi}$$

total $\boxed{468 \text{ mi}}$

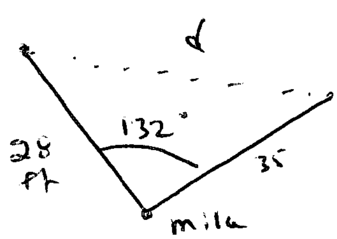
b) (2 points) Extra Credit: If the plane dropped one calculator every 5 feet, how many calculators were dropped on this trip?

$$468 \text{ mi} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ calculator}}{5 \text{ ft}} \approx \boxed{494{,}208 \text{ calculators!}}$$

BATTERIES NOT INCLUDED

3) (4 points) Giving away Halloween candy, Mike spots two children running blissfully through his muddy yard. Looking at one child, who is 28 feet away, Mike turns his head 132° to see the other child, who is 35 feet away. At that exact moment, how far are the children from each other?

$$d = \sqrt{28^2 + 35^2 - 2(28)(35)\cos 132°}$$



$$\approx \boxed{58 \text{ ft apart}}$$

4) (2 points) Concerning the given information of a triangle, how do you know when to use the Law of Sines versus the Law of Cosines?

Depending on which has the best Halloween candy.

17/19

5) (3 points) Find the trigonometric form of the complex number $5+5\sqrt{3}i$:

$r = \sqrt{5^2+(5\sqrt{3})^2} = \sqrt{100} = 10$

$\cos\theta = \frac{a}{r} = \frac{5}{10} = \frac{1}{2}$

$\sin\theta = \frac{b}{r} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$ $\Big\}\ \theta = \pi/3$

$\boxed{10\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

6) (3 points) Find the standard form of the complex number $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$= \sqrt{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \boxed{1+i}$

7) (4 points each) For the complex numbers $z_1 = 5+5\sqrt{3}i$ and $z_2 = 1+i$, find the following, using the trigonometric forms and the formula $z_1 \times z_2 = r_1 \times r_2\left[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right]$ for part *a* and $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right]$ for part *b*. ~~Write in standard form.~~ can't do it nicely

a) $z_1 \times z_2$

$= 10\sqrt{2}\left(\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)\right)$

$= 10\sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$

b) $\frac{z_1}{z_2}$

$= \frac{10}{\sqrt{2}}\left(\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\right)$

$= 5\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

8) (4 points part *a*; 6 points part *b*) For the complex number $-8i = 8(\cos 270° + i\sin 270°)$, find the following. For part *a*, use the formula $(a+bi)^n = r^n\left[\cos(n\theta)+i\sin(n\theta)\right]$. For part *b*, use the formula $(a+bi)^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n}+\frac{360°}{n}\cdot k\right)+i\sin\left(\frac{\theta}{n}+\frac{360°}{n}\cdot k\right)\right]$. Write answers in standard form.

a) $(-8i)^4$

$= 8^4\left(\cos(270°\cdot 4) + i\sin(270°\cdot 4)\right)$

$= 4096(\cos 1080° + i\sin 1080°)$

1080° is coterminal to 0°

$= 4096(\cos 0° + i\sin 0°)$

$= 4096(1+0i)$

$= \boxed{4096}$

b) The cube roots of $-8i$:

$8^{\frac{1}{3}}\left[\cos\left(\frac{270°}{3}+k\cdot\frac{360°}{3}\right) + i\sin\left(\frac{270°}{3}+k\cdot\frac{360°}{3}\right)\right]$

$k=0$ $\quad 2(\cos 90° + i\sin 90°) = \boxed{2i}$

$k=1$ $\quad 2(\cos 210° + i\sin 210°) = \boxed{-\sqrt{3}-i}$

$k=2$ $\quad 2(\cos 330° + i\sin 330°)$

$= \boxed{\sqrt{3}-i}$

JY

9) (4 points each) For the point $\left(1, \frac{\pi}{4}\right)$, find a different representation of the point in polar form that satisfies the following conditions:

a) $r < 0$ and $\theta > 0$

$\left(-1, \frac{5\pi}{4}\right)$

b) $r > 0$ and $\theta < 0$

$\left(1, -\frac{7\pi}{4}\right)$

c) $r > 0$ and $\theta > 0$

$\left(1, \frac{9\pi}{4}\right)$

infinitely many answers

10) (4 points each) Convert as stated:

a) $\left(\sqrt{2}, -\sqrt{2}\right)$ to polar:

$r = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = 2$

$\left.\begin{array}{l} \cos\theta = \frac{\sqrt{2}}{2} \\ \sin\theta = -\frac{\sqrt{2}}{2} \end{array}\right\} \theta = \frac{7\pi}{4}$ or $315°$

$\left(2, \frac{7\pi}{4}\right)$

b) $\left(9, \frac{11\pi}{6}\right)$ to rectangular:

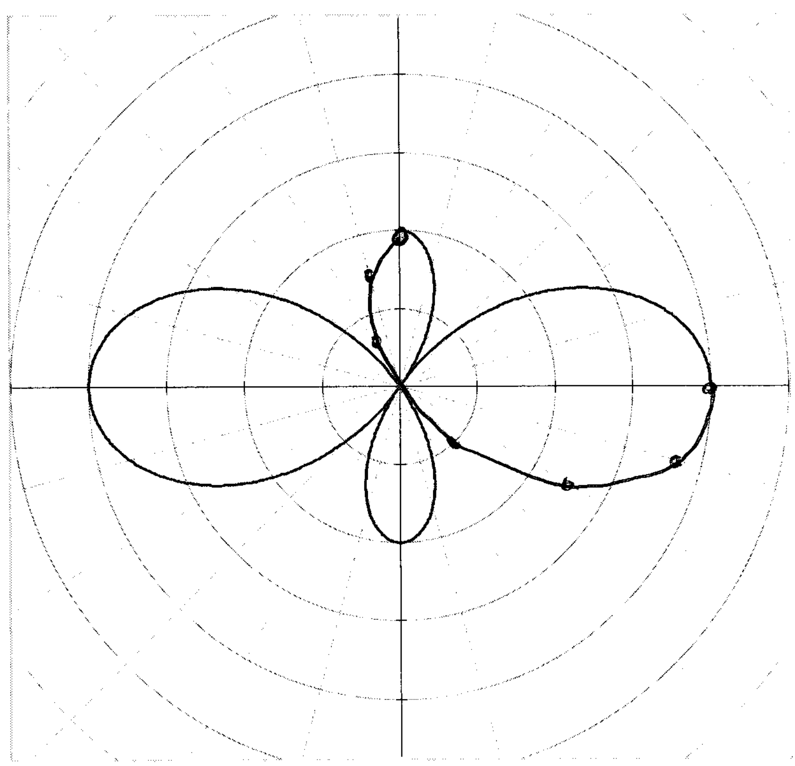
$x = 9\cos\frac{11\pi}{6} = \frac{9\sqrt{3}}{2}$

$y = 9\sin\frac{11\pi}{6} = -\frac{9}{2}$

$\left(\frac{9\sqrt{3}}{2}, -\frac{9}{2}\right)$

11) (4 points) Sketch a graph of $r = 3\cos(2\theta) + 1$. Part of the graph has been done for you. Use the values from $270°$ to $360°$ to finish the graph. Round to one decimal.

θ	r
270°	−2
285°	−1.6
300°	−0.5
315°	1
330°	2.5
345°	3.6
360°	4



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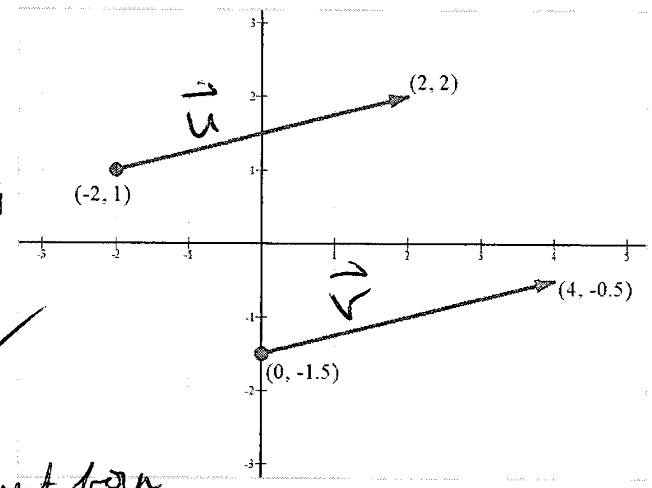
12) (4 points) For the given vectors, determine algebraically if they are equivalent:



$m_{\vec{u}} = \frac{2-1}{2-(-2)} = \frac{1}{4}$ $\quad m_{\vec{v}} = \frac{-0.5-(-1.5)}{4-0} = \frac{1}{4}$

$|\vec{u}| = \sqrt{(2-(-2))^2+(2-1)^2} = \sqrt{17}$

$|\vec{v}| = \sqrt{(4-0)^2-(-0.5-(-1.5))^2} = \sqrt{17}$ ✓

$\therefore \vec{u} = \vec{v}$ or shortcut both are $\langle 4,1\rangle$ in component form

13) (3 points each) Let $\vec{u} = \langle 3,2\rangle$ and $\vec{v} = \langle 8,-3\rangle$. Find and simplify:

a) $2\vec{u} - 4\vec{v}$

$2\vec{u} = \langle 6,4\rangle$

$-4\vec{v} = \langle -32,12\rangle$

$+$

$\langle -26, 16\rangle$

b) $|2\vec{u} - 4\vec{v}|$

$\sqrt{(-26)^2+(16)^2}$

$= \sqrt{932}$

$= 2\sqrt{233}$ ick!

c) The unit vector in the same direction as $2\vec{u} - 4\vec{v}$:

$\left\langle \frac{-26}{2\sqrt{233}}, \frac{16}{2\sqrt{233}} \right\rangle$

$\left\langle \frac{-13\sqrt{233}}{233}, \frac{8\sqrt{233}}{233} \right\rangle$ $(\text{ick})^2$

d) $\vec{u}\cdot\vec{v}$:

$= 3(8)+2(-3)$

$= 24-6 = \boxed{18}$

e) The angle between the vectors $\vec{u}$ and $\vec{v}$. Round to two decimal places. Use the formula $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$

$\theta = \cos^{-1}\left(\frac{18}{\sqrt{3^2+2^2}\cdot\sqrt{8^2+(-3)^2}}\right) \approx 54.25°$

$\sqrt{13}\cdot\sqrt{73}$

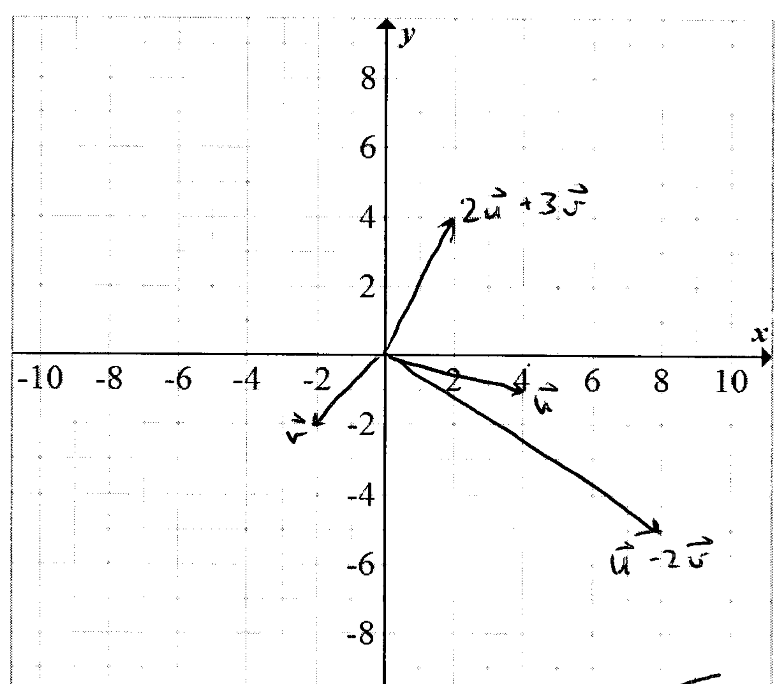
14) (4 points) Given the vector $\vec{u} = \langle 4,-1\rangle$ and $\vec{v} = \langle -2,2\rangle$, draw and label the vectors $\vec{u}, \vec{v}$, $\vec{u} - 2\vec{v}$, and $2\vec{u} + 3\vec{v}$:

✓

$\langle 4,-1\rangle + \langle 4,-4\rangle = \langle 8,-5\rangle$

$2\vec{u} + 3\vec{v} = \langle 8,-2\rangle + \langle -6,6\rangle$

$= \langle 2,4\rangle$



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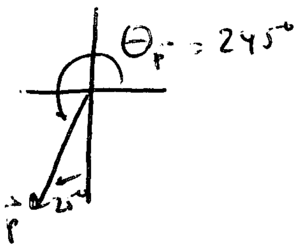
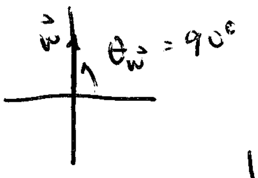
15) (8 points) An airplane travels on a bearing of $S25°W$ with an airspeed of 630 mph. A wind is blowing from due **south** at a speed of 55 mph. Find the ground speed and bearing of the plane using the formula $\vec{v} = |\vec{v}|(\cos\theta\vec{i} + \sin\theta\vec{j})$. Round only the final answer to two decimal places:



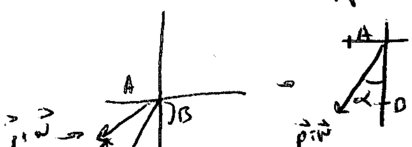
$\vec{p} = 630(\cos 245°\vec{i} + \sin 245°\vec{j})$

$\vec{w} = 55(\cos 90°\vec{i} + \sin 90°\vec{j})$



$\vec{p} + \vec{w} = \overbrace{[630\cos 245° + 55\cos 90°]}^{A}\vec{i} + \underbrace{[630\sin 245° + 55\sin 90°]}_{B}\vec{j}$

$|\vec{p} + \vec{w}| = \sqrt{A^2 + B^2} \approx 580.62$ mph



$\alpha = \tan^{-1}\left(\frac{A}{B}\right) \approx 27.29°$ $S27.29°W$

16) (4 points) A large, unattended child, pulls a wagon with a force of 15.5 lbs for 800 ft. The handle makes a $52°$ angle to the horizontal. How much work is done by the child in terms of foot-pounds? Use the formula $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$. Round only the final answer to two decimal places:

$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

$= 15.5 \cdot 800 \cdot \cos 52°$

$= 7634.2$ ft-lb

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