1) (5 points each) Simplify.

a) $\dfrac{\sin^2 2\alpha}{\sin^2 \alpha} = \dfrac{(2\sin\alpha\cos\alpha)^2}{\sin^2\alpha}$

$= \dfrac{4\sin^2\alpha\cos^2\alpha}{\sin^2\alpha} = \boxed{4\cos^2\alpha}$

b) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$= \cos 2x \cdot 1$

$= \boxed{\cos 2x}$

2) (4 points each) Find the exact value of $\tan\dfrac{13\pi}{12}$ using…

a) A Sum or Difference Formula:

$\tan\left(\frac{13\pi}{12}\right) = \tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$

$= \dfrac{\tan\frac{3\pi}{4} + \tan\frac{\pi}{3}}{1 - \tan\frac{3\pi}{4}\tan\frac{\pi}{3}} = \dfrac{-1+\sqrt{3}}{1+1\cdot\sqrt{3}}$

$= \boxed{\dfrac{-1+\sqrt{3}}{1+\sqrt{3}}}$

If rationalized, we get the answer in 2b.

b) A Half Angle Formula:

$\tan\left(\frac{13\pi}{12}\right) = \tan\left(\frac{\frac{13\pi}{6}}{2}\right) = \dfrac{1-\cos\frac{13\pi}{6}}{\sin\frac{13\pi}{6}}$

$= \dfrac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{2-\sqrt{3}}$

3) (3 points each) Using only the answers in 2a or 2b, **explain** how you would find the answer to the following. **Do not use formulas!**

a) $\tan\dfrac{\pi}{12}$

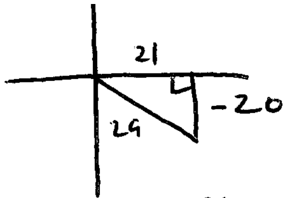
It'd be the same answer because tangent has the same value in Q I & III.

b) $\tan\dfrac{23\pi}{12}$

It'd be the negative answer because tangent has a negative value in Q II & IV.

4) (5 points) Simplify the expression: $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} = \dfrac{\cancel{(\sec^2 x + \tan^2 x)}(\sec^2 - \tan^2 x)}{\cancel{\sec^2 x + \tan^2 x}}$

$= \sec^2 x - \tan^2 x = \boxed{1}$



5) (4 points each) Given that $\cos\beta = \frac{21}{29}$ and that β is in Quadrant IV, find the exact value for…

a) $\sin(2\beta)$:

$2\left(-\frac{20}{29}\right)\left(\frac{21}{29}\right)$

-2 $\boxed{-\frac{840}{841}}$

b) $\cos(2\beta)$: $\left(\frac{21}{29}\right)^2 - \left(-\frac{20}{29}\right)^2$

$\boxed{\frac{41}{841}}$

c) $\tan(2\beta)$:

$\boxed{-\frac{840}{41}}$

d) The quadrant that 2β resides in and why:

Q IV

sine, tangent <0
cosine >0

6) (4 points a – c, 6 points ~~a~~ d) Simplify by finding the exact value:

a) $\sin^{-1}\left(\sin\frac{\pi}{12}\right)$

$= \boxed{\frac{\pi}{12}}$

b) $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \cos\left(-\frac{\pi}{6}\right)$

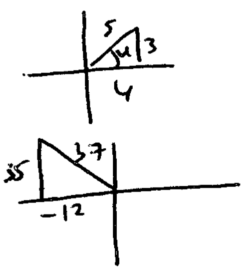
$= \boxed{\frac{\sqrt{3}}{2}}$

c) $\cos^{-1}\left(\cos\left(-\frac{5\pi}{4}\right)\right)$

$= \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$= \boxed{\frac{3\pi}{4}}$

d) $\sin\left(\overbrace{\sin^{-1}\frac{3}{5}}^{u}+\overbrace{\cos^{-1}\left(-\frac{12}{37}\right)}^{v}\right)$

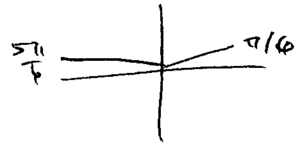


$= \sin\left(\sin^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}-\frac{12}{37}\right) + \cos\left(\sin^{-1}\frac{3}{5}\right)\sin\left(\cos^{-1}-\frac{12}{37}\right)$

$= \left(\frac{3}{5}\right)\left(-\frac{12}{37}\right) + \left(\frac{4}{5}\right)\left(\frac{35}{37}\right)$

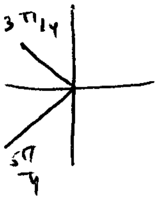
$= \boxed{\frac{104}{185}}$

34



7) (6 points each) Solve the equation for the variable:

a) $\sqrt{2}\cos x + 1 = 0$ (Hint: got k?)



$\cos x = -\frac{\sqrt{2}}{2}$

$$\boxed{\begin{aligned} x &= \frac{3\pi}{4} + 2\pi k \\ x &= \frac{5\pi}{4} + 2\pi k \end{aligned} \quad k \in \mathbb{Z}}$$

b) $\sin(2x) = \frac{1}{2}$ on $[0, 2\pi)$

Let $\theta = 2x$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} + 2\pi k$

$\theta = \frac{5\pi}{6} + 2\pi k$

$\rightarrow x = \frac{\pi}{12} + \pi k$

$x = \frac{5\pi}{12} + \pi k$

$k = 0 \quad k = 1$

$$\boxed{\left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\right\}}$$

8) (6 points) Fill in the blank using interval notation:

	$\sin x$*	$\cos x$*	$\tan x$*	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain	c	o	f	f	e	e
Range	i	s	g	o	o	d

*Write the domain restrictions for these three functions.

9) Short answer:

a) (4 points) Explain why $\sin\left(\sin^{-1}\frac{1}{2}\right) = \frac{1}{2}$ but $\sin^{-1}\left(\sin\frac{7\pi}{4}\right) \neq \frac{7\pi}{4}$.

cuz

b) Extra credit: (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

whatever

c) Extra credit: (2 points) Referring to number 5, find the exact value for $\sin(4\beta)$:

$$\boxed{-\frac{68{,}880}{707281}}$$

$\frac{22}{26}$