MATH 1540 Fall 2018 **Mike McCraith**
Exam 2 **October 23, 2018**

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

- **Write your name below on the space provided.**
- **This test has a total of 6 pages.**
- **Work the problem in the space provided. If you need more space, write on the back of the test.**
- **To insure maximum credit, show your work. In general, full credit will not be given for unsupported answers.**
- **Look only at your test. Don't give me the impression that you are cheating.**
- **Be sure to write neatly. If I cannot read what was written, do not expect the problem to be graded.**
- **If you finish early, go over the test again.**

Good luck!

Number	Maximum	Score
1	10	
2	8	
3	6	
4	5	
5	16	
6	18	
7	12	
8	6	
9	4/8	
10 TH	15	
Total	100	

Name _____

Circle your final answers

tear off the last two sheets

1) (5 points each) Simplify.

a) $\dfrac{\sin^2 2\alpha}{\sin^2 \alpha}$

b) $\cos^4 x - \sin^4 x$

2) (4 points each) Find the exact value of $\tan\dfrac{13\pi}{12}$ using…

a) A Sum or Difference Formula:

b) A Half Angle Formula:

3) (3 points each) Using only the answers in 2a or 2b, **<u>explain</u>** how you would find the answer to the following. **Do not use formulas!**

a) $\tan\dfrac{\pi}{12}$

b) $\tan\dfrac{23\pi}{12}$

4) (5 points) Simplify the expression: $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x}$

5) (4 points each) Given that $\cos\beta = \frac{21}{29}$ and that β is in Quadrant IV, find the exact value for…

a) $\sin(2\beta)$:

b) $\cos(2\beta)$:

c) $\tan(2\beta)$:

d) The quadrant that 2β resides in and why:

6) (4 points a – c, 6 points 6) Simplify by finding the exact value:

a) $\sin^{-1}\left(\sin\frac{\pi}{12}\right)$

b) $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

c) $\cos^{-1}\left(\cos\left(-\frac{5\pi}{4}\right)\right)$

d) $\sin\left(\sin^{-1}\frac{3}{5}+\cos^{-1}\left(-\frac{12}{37}\right)\right)$

7) (6 points each) Solve the equation for the variable:

a) $\sqrt{2}\cos x + 1 = 0$ (Hint: got *k*?)

b) $\sin(2x) = \frac{1}{2}$ on $[0, 2\pi)$

8) (6 points) Fill in the blank using interval notation:

	$\sin x$ *	$\cos x$ *	$\tan x$ *	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain						
Range						

*Write the domain restrictions for these three functions.

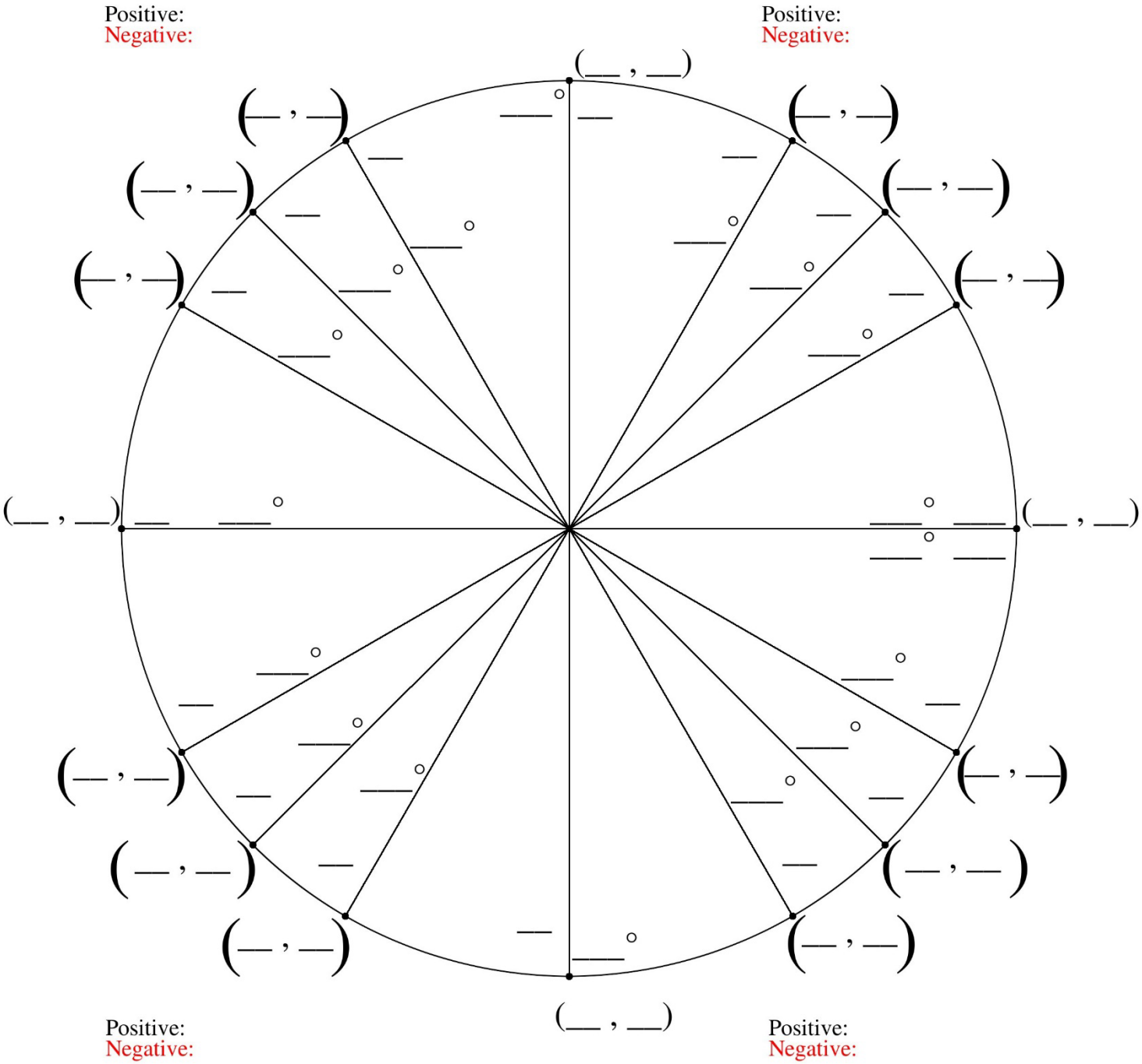
9) Short answer:

a) (4 points) Explain why $\sin\left(\sin^{-1}\frac{1}{2}\right) = \frac{1}{2}$ but $\sin^{-1}\left(\sin\frac{7\pi}{4}\right) \neq \frac{7\pi}{4}$.

b) Extra credit: (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

c) Extra credit: (2 points) Referring to number 5, find the exact value for $\sin(4\beta)$:

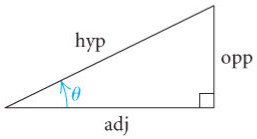
This page is not for credit. It is provided in case you need it to fill in and reference while you take the exam.



Trigonometry

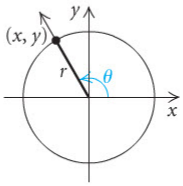
Trigonometric Functions

Acute Angles



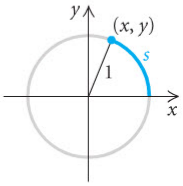
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \csc\theta = \frac{\text{hyp}}{\text{opp}},$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \sec\theta = \frac{\text{hyp}}{\text{adj}},$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}}, \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Any Angle



$$\sin\theta = \frac{y}{r}, \quad \csc\theta = \frac{r}{y},$$
$$\cos\theta = \frac{x}{r}, \quad \sec\theta = \frac{r}{x},$$
$$\tan\theta = \frac{y}{x}, \quad \cot\theta = \frac{x}{y}$$

Real Numbers



$$\sin s = y, \quad \csc s = \frac{1}{y},$$
$$\cos s = x, \quad \sec s = \frac{1}{x},$$
$$\tan s = \frac{y}{x}, \quad \cot s = \frac{x}{y}$$

Basic Trigonometric Identities

$$\sin(-x) = -\sin x,$$
$$\cos(-x) = \cos x,$$
$$\tan(-x) = -\tan x,$$

$$\tan x = \frac{\sin x}{\cos x},$$
$$\cot x = \frac{\cos x}{\sin x},$$

$$\csc x = \frac{1}{\sin x},$$
$$\sec x = \frac{1}{\cos x},$$
$$\cot x = \frac{1}{\tan x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1,$$
$$1 + \cot^2 x = \csc^2 x,$$
$$1 + \tan^2 x = \sec^2 x$$

Identities Involving $\pi/2$

$$\sin(\pi/2 - x) = \cos x,$$
$$\cos(\pi/2 - x) = \sin x, \quad \sin(x \pm \pi/2) = \pm\cos x,$$
$$\tan(\pi/2 - x) = \cot x, \quad \cos(x \pm \pi/2) = \mp\sin x$$

Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v,$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v,$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x,$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1,$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}, \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}},$$
$$\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$