1) (6 points) Fill in the information for the parabola $(x-4)^2 = 8(\overset{y}{\cancel{x}}+1)$:

i) $h =$ 4

ii) $k =$ -1

iii) $p =$ 2

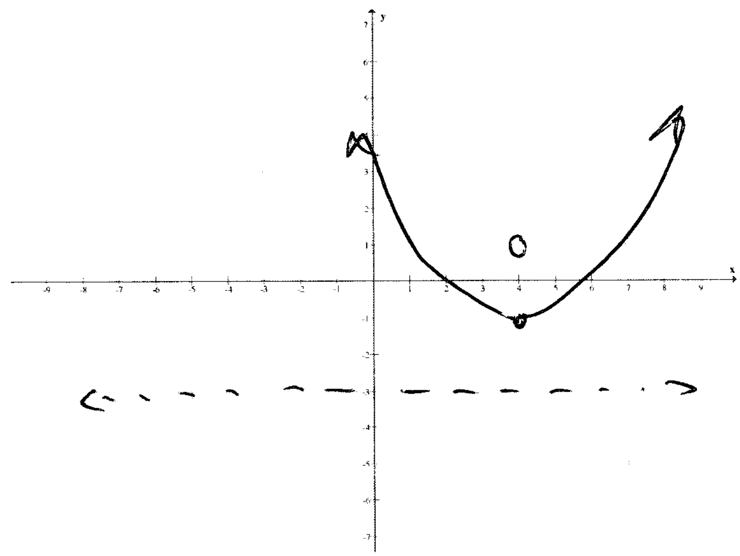
Write iv and v as ordered pairs:

iv) Vertex (4, -1)

v) Focus (4, 1)

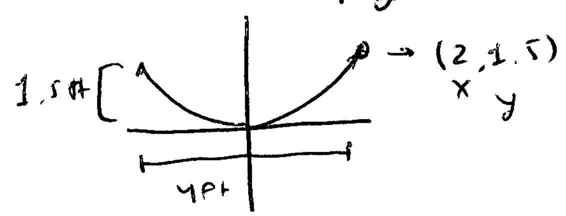
vi) Directrix $y = -3$

vi) Sketch the graph:



2) (4 points) While on stage, Mike notices that the spotlight on him has a parabolic mirror that is 4 feet wide and 1.5 feet deep. If the bulb for the spotlight is at the focus of the parabola, how far, in inches, from the vertex of the parabola is the bulb located?

$x^2 = 4py$

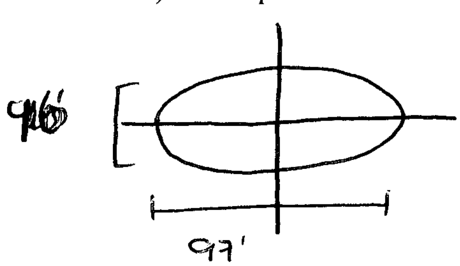


$2^2 = 4 \cdot p \cdot 1.5 \Rightarrow p = \frac{2}{6} = \frac{2}{3}$ ft

$\boxed{= 8 \text{ in}}$

3) (3 points each) Statuary Hall, also known as the Whispering Gallery, is an elliptical room in the United States Capitol in Washington D.C. where a person standing at one focus of the room can hear even a whisper spoken by a person standing at the other focus. Statuary Hall is 46 feet wide and 97 feet long. Assuming a horizontal ellipse, find…

a) The equation of the ellipse for the room:



$A = \frac{97}{2}$ $B = 23$

$$\frac{x^2}{(\frac{97}{2})^2} + \frac{y^2}{23^2} = 1$$

$23^2 \rightarrow$ or 529

or $\frac{9409}{4}$ or 2352.25

b) The location of the foci from the center of the room. Round to two decimal places:

$c = \sqrt{(\frac{97}{2})^2 - 23^2}$

$\approx$ 42.7 ft

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4) (8 points) Fill in the information for the hyperbola $\frac{y^2}{25}-\frac{x^2}{144}=1$:

i) $h =$ 0

ii) $k =$ 0

iii) $a =$ 5

iv) $b =$ 12

v) $c =$ 13

Write as ordered pairs:

vi) Center (0,0)

vii) Vertices (0,5) (0,-5)

viii) Co-Vertices (12,0) (-12,0)

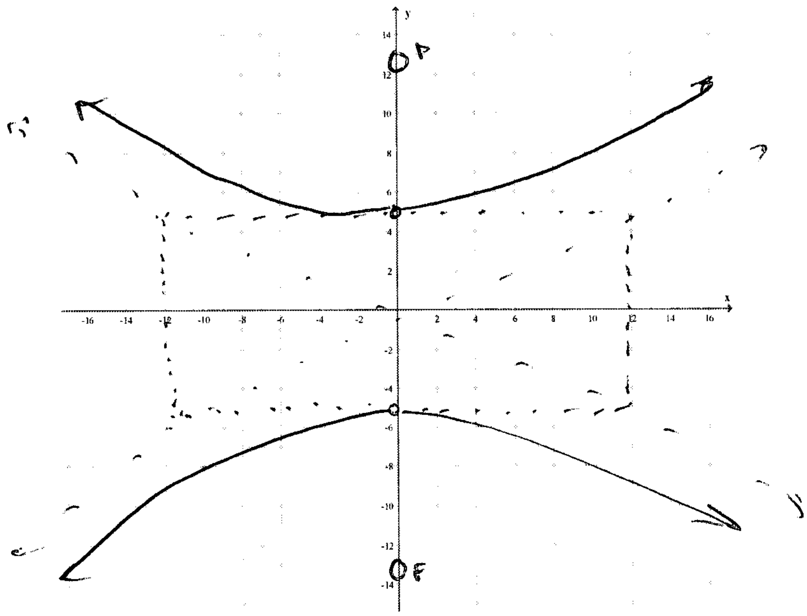
ix) Foci (0,13) (0,-13)

Write in slope-intercept form:

x) Asymptotes $y = \frac{5}{12}x$

$y = -\frac{5}{12}x$

vi) Sketch the graph:



5) (5 points each) Solve the following systems. For part *a*, write your answer in ordered pairs. For part *b*, shade your final in the darkest:

a) $\begin{cases} 4x^2+y^2=16 \\ x^2-y^2=4 \end{cases}$

ADD

$5x^2 = 20$

$x^2 = 4$

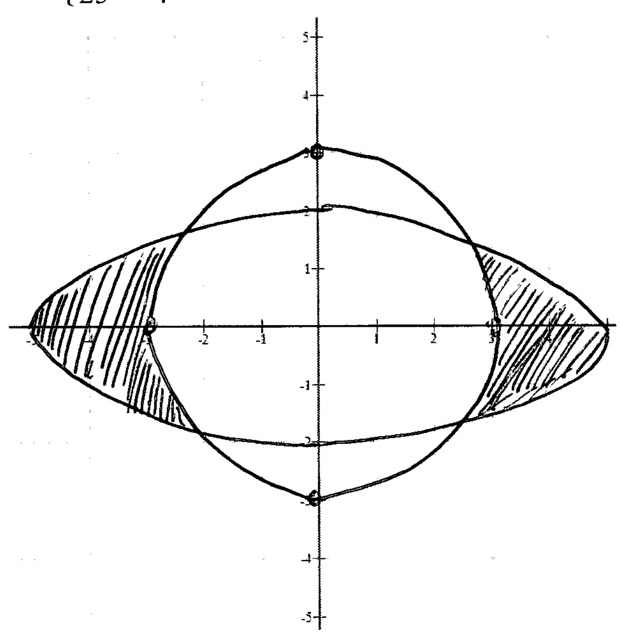
$x = \pm 2$

$4(2)^2 + y^2 = 16 \Rightarrow y = 0$

$4(-2) + y^2 = 16 \Rightarrow y = 0$

(2,0) (-2,0)

b) $\begin{cases} x^2+y^2 \geq 9 & \text{circle} \\ \frac{x^2}{25}+\frac{y^2}{4} \leq 1 & \text{ellipse} \end{cases}$



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6) (2 points) What is a sequence?

A fun card game for the whole family!

7) (6 points each) Find the first five terms of the following sequences. Determine if they are arithmetic, geometric, or neither. If it is arithmetic, determine the common difference. If it is geometric, determine the common ratio.

a) $\{2(-3)^n\}$

$n=1 \quad 2(-3)^1 = -6$
$n=2 \quad 2(-3)^2 = 18$
$n=3 \quad 2(-3)^3 = -54$
$n=4 \quad 2(-3)^4 = 162$
$n=5 \quad 2(-3)^5 = -486$

geometric
$r=-3$

b) $a_1 = 3,\ a_{n+1} = a_n + \frac{1}{2},\ n \geq 1$

$a_1 = 3$
$a_2 = 3 + \frac{1}{2} = 3.5$
$a_3 = 3.5 + \frac{1}{2} = 4$
$a_4 = 4 + \frac{1}{2} = 4.5$
$a_5 = 4.5 + \frac{1}{2} = 5$

arithmetic
$d = 1/2$

8) (4 points) Find the sum $\sum_{k=1}^{6} \frac{k}{3}$. Be sure to write out the individual terms:

$= \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \frac{4}{3} + \frac{5}{3} + \frac{6}{3} = 7$

9) (4 points each) Write in sigma notation:

a) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots + \frac{49}{50}$

$\sum_{k=1}^{49} \frac{k}{k+1}$

b) $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \ldots + \frac{1}{4{,}096}$

$\left(-\frac{1}{4}\right)^0 \quad \left(-\frac{1}{4}\right)^1 \quad \left(-\frac{1}{4}\right)^2 \quad \left(-\frac{1}{4}\right)^3 \quad \longrightarrow \left(-\frac{1}{4}\right)^6$

$\sum_{k=0}^{6} \left(-\frac{1}{4}\right)^k$

10) (3 points each) For the arithmetic sequence $-4, -1, 2, 5, 8 \ldots$, find and simplify…

a) a_n using $a_n = a_1 + (n-1)d$

$a_n = -4 + (n-1) \cdot 3$
$= 3n - 7$

b) a_{412}:

$a_{412} = 3(412) - 7$
$= 1229$

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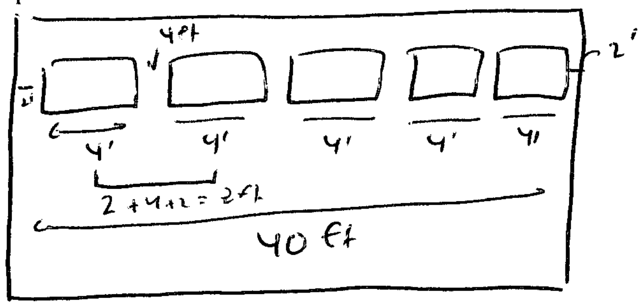
Nom Nom

11) (4 points each) Mike decides to hang some photos of his favorite pies on a 40 foot wall in his office. The 5 frames are 4 feet long each. He wants the first and last frames to be in from the corner by 2 feet and equal spacing in between each frame.

a) Draw a complete picture for this scenario:



Space inbetween

$$\frac{40-5\cdot4-2\cdot2}{4}$$

$= 4$ ft

b) Assuming the nails will go in the exact horizontal center of the frame, where should Mike put the nails into the wall? Start your count from the left corner:

4', 12', 20', 28', 36'

12) (4 points each) Evaluate the sums using either formula: $S_n=\frac{n}{2}\left(2a_1+(n-1)d\right)$ or $S_n=\frac{n}{2}\left(a_1+a_n\right)$.

Write answers as improper fractions as needed:

a) $\sum_{k=1}^{70}(4k-10)$

$4(1)-10 \qquad 4(70)-10$

$$=\frac{70}{2}\left(-6+270\right)$$

$= 9240$

b) $\sum_{k=12}^{48}\left(\frac{5k+10}{4}\right)$

$48-12+1 \qquad \frac{5(12)+10}{4} \qquad \frac{5(48)+10}{4}$

$$=\frac{37}{2}\left(\frac{35}{2}+\frac{125}{2}\right)$$

$= 1480$

13) (3 points each) For the geometric sequence $\frac{2}{9},\frac{2}{3},2,6,\ldots$, find the following given $a_n=a_1r^{n-1}$:

a) a_n

$$=\frac{2}{9}(3)^{n-1}$$

b) a_{15}

$$a_{15}=\frac{2}{9}(3)^{15-1}=1{,}062{,}882$$

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14) (4 points each) Using the formulas $S_n = a_1 \frac{1-r^n}{1-r}$ and $S_\infty = \frac{a_1}{1-r}$ (respectively), find by writing answers as an improper fraction…

a) $8+2+\frac{1}{2}+\frac{1}{8}+\ldots+\frac{1}{128}$

geometric

$8(\frac{1}{4})^0 + 8(\frac{1}{4})^1 + 8(\frac{1}{4})^2 + \ldots + 8(\frac{1}{4})^5$

from 0 to 5 is $n=6$

$$S_6 = 8 \cdot \frac{1-(\frac{1}{4})^6}{1-\frac{1}{4}} = \boxed{\frac{1365}{128}}$$

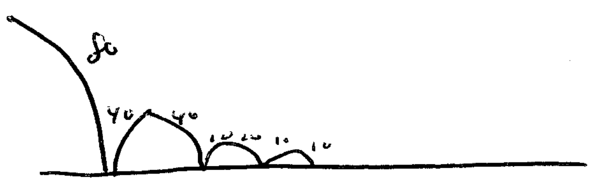
b) $8+2+\frac{1}{2}+\frac{1}{8}+\ldots$

geometric

$a_1 = 8 \quad r = \frac{1}{4}$

$$= \frac{8}{1-\frac{1}{4}} = \frac{32}{3}$$

15) (4 points) A ball is dropped from a height of 80 feet and always rebounds ½ of the distance fallen. How far does the ball travel vertically before coming to a stop? In your answer be sure to use the formula $S_\infty = \frac{a_1}{1-r}$.



$$80 + 2\left(\frac{40}{1-\frac{1}{2}}\right) = 240\text{ft}$$

or

$$\frac{80}{1-\frac{1}{2}} + \frac{40}{1-\frac{1}{2}}$$

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