1) (3 points each) Find the inverse of the following functions:

a) $\{(6,-2),(-5,8),(0,-9),(16,1)\}$

$\{(-2,6),(8,-5),(-9,0),(1,16)\}$

b) $f(x)=\frac{x+7}{x-5}$

① $y=\frac{x+7}{x-5}$

② $x=\frac{y+7}{y-5}$

③ $x(y-5)=y+7$

$xy-5x=y+7$

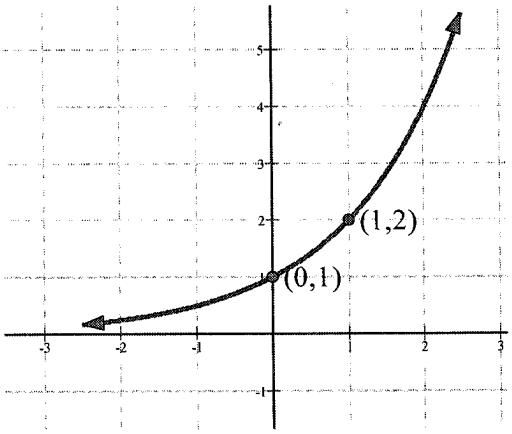
$xy-y=5x+7$

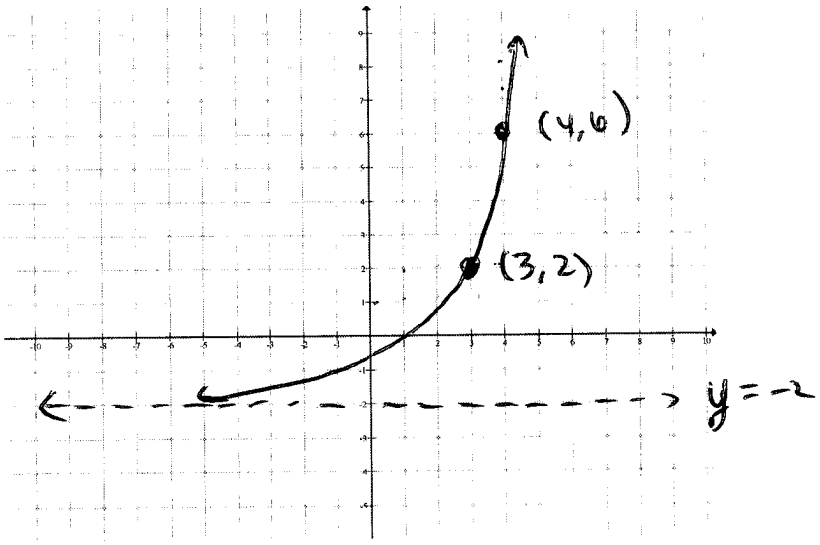
$y(x-1)=5x+7$

$y=\frac{5x+7}{x-1}$

④ $f^{-1}(x)=\frac{5x+7}{x-1}$

2) (3 points) Sketch the graph of the function $f(x)=4\cdot 2^{x-3}-2$. Be sure to label the transformed given points and asymptote:





① Right 3
② Vert. stretch by 4
③ Down 2

3) (4 points) Write the expression as one logarithm: $5\log x+9\log y-\frac{1}{3}\log z$

$=\log x^5+\log y^9-\log z^{1/3}$

$=\log\left(\frac{x^5y^9}{z^{1/3}}\right)$

4) (4 points) Given that $\log_a x=3$, $\log_a y=-4$, and $\log_a z=5$, find the value of $\log_a\sqrt{\frac{x^3y^5}{z^2}}$:

$=\log_a\left(\frac{x^3y^5}{z^2}\right)^{1/2}=\frac{1}{2}\log_a\left(\frac{x^3y^5}{z^2}\right)$ squishy!

$=\frac{1}{2}\left[\log_a x^3+\log_a y^5-\log_a z^2\right]=\frac{1}{2}\left[3\log_a x+5\log_a y-2\log_a z\right]=\frac{1}{2}\left[3\cdot 3+5(-4)-2(5)\right]$

$=\left|-\frac{21}{2}\text{ or }-10.5\right|$

5) (2 points) Short answer: Why are logarithms necessary?

They prevent people from only seeing the movie and never reading the book.

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6) (3 points each) Simplify completely:

a) $\log_{25}(5x)$

$= \log_{25} 5 + \log_{25} x$

$= \boxed{\frac{1}{2} + \log_{25} x}$

b) $\ln\left(\frac{e^9}{x^2}\right)$

$= \ln e^9 - \ln x^2$

$= \boxed{9 - 2\ln x}$

c) $\log_2 25 \times \log_5 8$

$= \frac{\log 25}{\log 2} \cdot \frac{\log 8}{\log 5} = \frac{\log 25}{\log 5} \cdot \frac{\log 8}{\log 2}$

$= \log_5 25 \cdot \log_2 8 = 2 \cdot 3 = \boxed{6}$

7) (5 points each) Solve for the variable.

a) $4^{3x+7} = 64$

$4^{3x+7} = 4^3$

$3x+7 = 3 \Rightarrow \boxed{x = \frac{-4}{3}}$

b) $5^{3x-2} = 35$

$\log_5 35 = 3x - 2$

$\boxed{\frac{\log_5 35 + 2}{3} = x}$

c) $\ln(4x-7) - \ln(x-2) = \ln(5)$

$\ln\left(\frac{4x-7}{x-2}\right) = \ln 5$

$\frac{4x-7}{x-2} = 5 \Rightarrow 4x-7 = 5x-10$

$\boxed{3 = x}$

d) $\log_2(5x+15) - \log_2(x+2) = 3$

$\log_2 \frac{5x+15}{x+2} = 3$

$2^3 = \frac{5x+15}{x+2} \Rightarrow 8x+16 = 5x+15$

$3x = -1$

$\boxed{x = -1/3}$

e) $e^x - 1 = 2e^{-x}$

multiply by e^x:

$(e^x)^2 - (e^x) = 2$

$(e^x)^2 - (e^x) - 2 = 0$

Let $u = e^x$

$u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$u = 2 \quad u = -1$

$e^x = 2$

$\boxed{x = \ln 2}$

$e^x = -1$

$\emptyset$

8) The video "My Cat Powers Up His Attacks" started to go viral last year. At 8 am, when the video was posted, there were 300 views. At noon, there 12,000 views. Assume that the number of views is growing exponential and use the formula $P(t) = P_0 e^{kt}$ where P is the number of views and t is the number of hours past 8 am.

a) (5 points) Determine the exact value for the growth rate k.

when $t=0$, $P(0) = 300$

when $t=4$, $P(4) = 12000$

$12000 = 300e^{k \cdot 4}$

$40 = e^{4k}$

$\ln 40 = 4k$

$\boxed{k = \frac{\ln 40}{4}}$

b) Extra Credit: During the exact time in hour:minute:second format when there was 60,000 views.

$60000 = 300e^{\frac{\ln 40}{4}t}$

$200 = e^{\frac{\ln 40}{4}t}$

$\ln 200 = \frac{\ln 40}{4} \cdot t$

$t = \frac{\ln 200}{\left(\frac{\ln 40}{4}\right)} \approx 5.745178103$ hours

$\times 60$

44.71065621 min

$\times 60$

$42.\ldots$ sec

5 hr, 44 min, 42 sec after 8 AM

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9) (1 measly point) Fill in the blank: John Jacob Jingleheimer Schmidt, a foreign exchange student from Norway, is in your math class. (Yes, in the future, you'll be teaching math—kudos: me.) He asks one day for you to pronounce $\log_4 12$ for him. You reply "Gladly, it's pronounced I can haz algebra?"

10) (10 points each) Solve the system $\begin{cases} x+2y+3z=4 \\ -2x-3y-6z=-3 \\ 3x+6y+10z=6 \end{cases}$ using the methods listed below. Be sure to show all necessary work where appropriate. Write answer as an ordered triple:

a) Elimination Method:

① multiply first eq by 2

$2x+4y+6z=8$

$-2x-3y-6z=-3$

$y = 5$

② multiply first eq by -3

$-3x-6y-9z=-12$

$3x+6y+10z=6$

$z=-6$

we can skip steps 3 & 4. woot.

$x+2(5)+3(-6)=4$

$x=12$

$(12, 5, -6)$

b) Gauss-Jordan Method:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ -2 & -3 & -6 & -3 \\ 3 & 6 & 10 & 6 \end{array}\right] \xrightarrow{\substack{2R_1+R_2 \to R_2 \\ -3R_1+R_3 \to R_3}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -6 \end{array}\right]$$

$$\xrightarrow{\substack{-2R_2+R_1 \to R_1 \\ -3R_3+R_1 \to R_1}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -6 \end{array}\right]$$

11) (1 point) Verify that you made absolutely sure that your answer to 10a is the same as in 10b by signing your name here Bob Q. You will not receive the credit if the work does not support the same answer.

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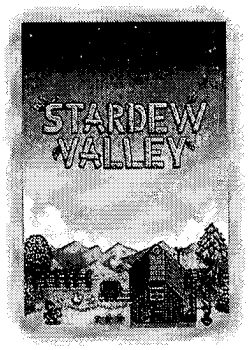
12) (6 points) **Set up <u>BUT DO NOT SOLVE</u>** the system associated with the following. **Be sure to name and define your variables**:



Play this game if you enjoy losing countless hours of your life in front of the TV.

It is rumored that next year, Nintendo will release an updated version of the Nintendo Switch and Mike is strongly considering buying one, despite already having a plethora of games to play. He also has a problem of buying more than one copy of the same game. At Target, Mike orders 2 copies of *Zelda*, 4 copies of *Super Mario Odyssey*, and 2 copies of *Stawdew Valley* and spends \$350.44. At Best Buy, Mike orders 4 copies of *Zelda*, 5 copies of *Super Mario Odyssey*, and 1 copy of *Stawdew Valley* and spends \$495.44. The price for one copy of *Stawdew Valley* is half of the other two games combined. Assume the prices for each game does not change depending on where the game was purchase. How ~~much~~ does each game cost?

Let x = cost of Zelda
y = " " SMO
z = " " SV

$$2x + 4y + 2z = 350.44$$
$$4x + 5y + 1z = 495.44$$
$$z = \tfrac{1}{2}(x+y) \quad \text{or} \quad -\tfrac{1}{2}x - \tfrac{1}{2}y + z = 0$$

13) (7 points each) Decompose into partial fractions:

a) $\dfrac{x+10}{x^2-4x-12} = \dfrac{A}{x-6} + \dfrac{B}{x+2}$

$(x-6)(x+2)$

$$x+10 = A(x+2) + B(x-6)$$
$$= (A+B)x + (2A-6B)$$

$$\begin{cases} A+B = 1 \\ 2A-6B = 10 \end{cases} \qquad A=2 \quad B=-1$$

$$\frac{2}{x-6} + \frac{-1}{x+2}$$

b) $\dfrac{x^2+2x+7}{(x^2+2)(x+1)} = \dfrac{Ax+B}{x^2+2} + \dfrac{C}{x+1}$

$$x^2+2x+7 = (Ax+B)(x+1) + C(x^2+2)$$
$$= Ax^2 + Ax + Bx + B + Cx^2 + 2C$$

$$\begin{cases} A \quad +C = 1 \\ A+B \quad = 2 \\ B+2C = 7 \end{cases}$$

$A=-1 \quad B=3 \quad C=2$

$$\frac{-x+3}{x^2+2} + \frac{2}{x+1}$$

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