1) (3 points) Solve for the variable: $x^4 + 34x^2 + 225 = 0$:

$(x^2+9)(x^2+25) = 0$

$x^2 = -9 \Rightarrow \boxed{x = \pm 3i}$

$x^2 = -25 \Rightarrow \boxed{x = \pm 5i}$

2) (2 points each) For the function $f(x) = 2x^2 + 6x - 5$, determine…

a) If it opens up or down. How do you know?

up, $a = 2 > 0$

b) The coordinates of the vertex:

$x = \frac{-6}{2(2)} = -\frac{3}{2}$ $\quad f(-\frac{3}{2}) = -\frac{19}{2}$

$\boxed{(-\frac{3}{2}, -\frac{19}{2})}$

c) The domain:

$\mathbb{R}$

d) The range:

$[-\frac{19}{2}, \infty)$

e) Interval of increase:

$(-\frac{3}{2}, \infty)$

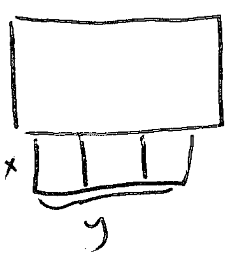
f) Interval of decrease:

$(-\infty, -\frac{3}{2})$

g) Based only on your answers to parts *a* and *b*, will it have *x*-intercepts? Why or why not?

vertex below x-axis, opens up. So there are x-int.

3) (5 points) Combining his love of farm life and coffee, Mike opens up a new coffeehouse called *Joe Joe Bahhh*. Next to the coffeehouse, he wishes to build three adjacent rectangular pens that all use the coffeehouse as an edge. He has 150 feet of fencing available. Assuming the edge along the coffeehouse needs no fencing, what should the dimensions of the enclosure be to maximize the area and what is the maximum area?



$4x + y = 150 \Rightarrow y = 150 - 4x$

$A = xy = x(150 - 4x) = -4x^2 + 150x$

$x = \frac{-150}{2(-4)} = \boxed{18.75 \text{ ft wide}}$ $\quad y = 150 - 4(18.75)$ $\boxed{75 \text{ feet long}}$

$18.75 \cdot 75 = \boxed{1406.25 \text{ ft}^2 \text{ max area}}$

4) (3 points each) Solve for the variable. Write ~~part b's answer in interval notation~~: Bow!

a) $x + \frac{12}{x-3} = 1 + \frac{4x}{x-3}$

multiply all terms by $x-3$

$x(x-3) + 12 = x - 3 + 4x$

$x^2 - 3x + 12 = 5x - 3$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$\cancel{x = 3}$ $\quad \boxed{x = 5}$

not in domain

b) $6|2x+1| - 7 = 10$

$|2x+1| = \frac{17}{6}$

$2x + 1 = \frac{17}{6}$ $\quad 2x + 1 = -\frac{17}{6}$

$\boxed{x = \frac{11}{12} \quad x = -\frac{23}{12}}$

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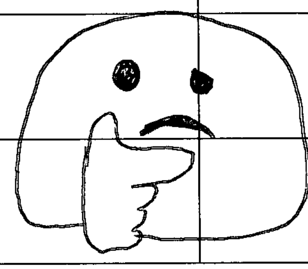
5) (2 points) Fill in the chart with a sketch of the location of the arrowheads:

	Even Degree	Odd Degree
Positive Leading Coefficient		
Negative Leading Coefficient		



6) (2 points each) Form a polynomial <u>function</u> of degree four that meets the following requirements. **Be sure to leave your answer in factored form**:

a) Has zeros at 5, -8, 6, and 12:

$f(x) = (x-5)(x+8)(x-6)(x-12)$

b) Has the same zeros and multiplicity as in part *a* but is a different function:

$g(x) = \underbrace{\frac{\pi^2 e}{\ln 6}}_{\text{any non zero real \#.}} \cdot (x-5)(x+8)(x-6)(x-12)$

c) Has zeros including $2-7i$ and $12+\sqrt{3}$:

$h(x) = (x-(2-7i))(x-(2+7i))(x-(12+\sqrt{3}))(x-(12-\sqrt{3}))$

7) (3 points *a;* 2 points others) Consider the functions $f(x) = 2x^3 + x^2 - 7x - 26$ and $g(x) = x^2 + 3x + 4$.

a) Divide $f(x)$ by $g(x)$ using long division:

$$
\begin{array}{r}
2x \quad - 5 \\
x^2+3x+4 \overline{)\, 2x^3 + x^2 - 7x - 26} \\
-(2x^3 + 6x^2 + 8x) \\
\hline
-5x^2 - 15x - 26 \\
-(-5x^2 - 15x - 20) \\
\hline
-6
\end{array}
$$

b) Based on your work in part *a*, was $g(x)$ a factor of $f(x)$? Why or why not?

NO remainder wasn't zero

c) What is the equation of the oblique asymptote of the rational function $y = \dfrac{2x^3 + x^2 - 7x - 26}{x^2 + 3x + 4}$?

$\boxed{y = 2x - 5}$

TS

8) (8 points each) Factor the polynomial completely by first listing the possible rational roots and then using synthetic division and your calculator.

a) $f(x)=3x^3+16x^2+15x-18$

$\frac{p}{q} = \pm 1, 2, 3, 6, 9, 18, \frac{1}{3}, \frac{2}{3}$

$$\begin{array}{r|rrrr} -3 & 3 & 16 & 15 & -18 \\ & & -9 & -21 & 18 \\ \hline -3 & 3 & 7 & -6 & \lfloor 0 \\ & & -9 & 6 & \\ \hline & 3 & -2 & \lfloor 0 & \end{array}$$

$\boxed{(x+3)^2(3x-2)}$

b) $g(x) = x^5 - 6x^4 - 3x^3 + 42x^2 + 2x - 60$

$\frac{p}{q} = \pm 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$

$$\begin{array}{r|rrrrrr} -2 & 1 & -6 & -3 & 42 & 2 & -60 \\ & & -2 & 16 & -26 & -32 & 60 \\ \hline 3 & 1 & -8 & 13 & 16 & -30 & \lfloor 0 \\ & & 3 & -15 & -6 & 30 & \\ \hline 5 & 1 & -5 & -2 & 10 & \lfloor 0 & \\ & & 5 & 0 & -10 & & \\ \hline & 1 & 0 & -2 & \lfloor 0 & & \end{array}$$

$\rightarrow x^2-2 = (x+\sqrt{2})(x-\sqrt{2})$

$\boxed{(x+2)(x-3)(x-5)(x+\sqrt{2})(x-\sqrt{2})}$

9) (3 points each) Using your factorized result from 8a, complete the following for the function $f(x)=3x^3+16x^2+15x-18$:

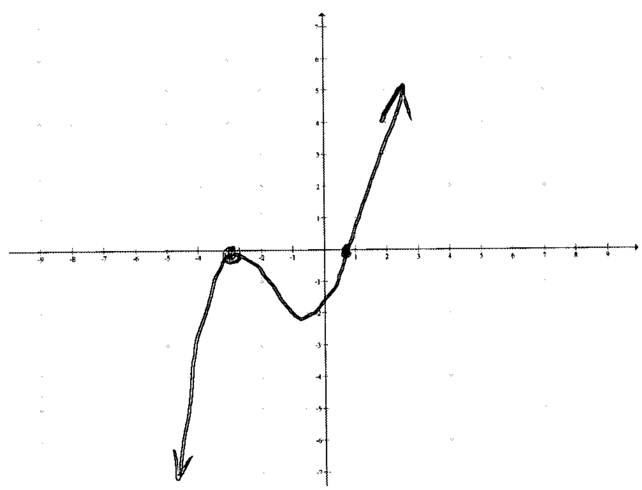
a) What is the leading term and in which quadrants will the arrowheads end up?

$3x^3$ Q III & I

b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
-3	2	Touch
$2/3$	1	Cross

c) Sketch the graph based on parts *a* and *b*:



10) (2 points each blank) Fill in the blank:

a) If *c* is a zero of a function *f*, then $f(c)=$ duck, and duck is a factor.

b) Numbers not in the domain of a rational function lead to goose.

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11) (4 points each) For the function $f(x)=\dfrac{2x+2}{x^2-1}$, find…

a) The domain:

$x^2-1=0$ $(x+1)(x-1)=0$

$\boxed{x \neq \pm 1}$

b) The intercepts (if any):

x-int

$2x+2=0$

$x=-1$

not in domain. No x-int.

y-int

$f(0)=\frac{2}{-1}=-2$

$(0,-2)$

c) Any vertical asymptotes and holes:

$x=1$

$2(1)+2 \neq 0$

$X=1$ VA

$x=-1$

$2(-1)+2=0$ Hole

$\dfrac{2\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$

$\dfrac{2}{x-1}$ @ $x=-1$ $\dfrac{2}{-1-1}=-1$

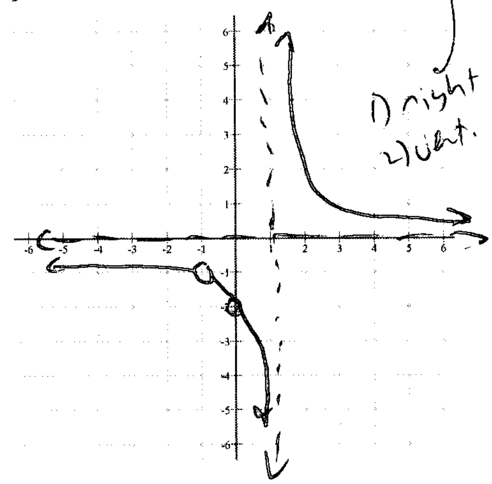
$\boxed{(-1,-1) \text{ Hole}}$

d) Any horizontal or oblique asymptotes:

HA

$y=0$

e) Sketch a graph using the above information.
HINT: Consider transformations based on the simplified version of the function!



1) right 1
2) vert. stretch by 2.

12) (3 points each) Solve for the variable. Write answer in interval notation:

a) $x^3-x^2-6x \leq 0$

$x(x-3)(x+2)=0$

$x=0,3,-2$

$-$ $+$ $-$ $+$

-2 0 3

$(-\infty,-2]\cup[0,3]$

b) $\dfrac{x-3}{x+2} \geq 0$

$+$ $-$ $+$

-2 3

$(-\infty,-2)\cup[3,\infty)$

13) Extra Credit: (3 points each) Short answer. Clearly explain how to find the following algebraically:

a) Vertical Asymptotes and Holes:

put the lime in the coconut

b) Horizontal and Oblique Asymptotes:

Call the doctor wake 'im up

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