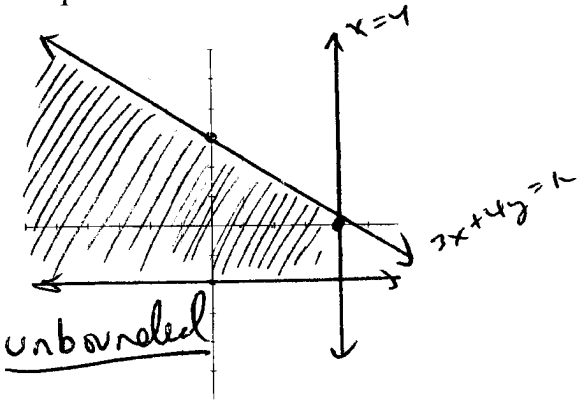
1) (6 points) Sketch the following system of linear inequalities. **Determine if the solution is bounded or unbounded.**

$$\begin{cases} 3x+4y \le 12 \\ x \le 4 \\ y \ge -2 \end{cases}$$



2) Solve the following Maximization LP using the corner point method:

In Karla's garden shop, she makes two kinds of mixtures for planting: gardening mixture and potting mixture. A package of gardening mixture requires 2 lb of soil, 1 lb of peat moss, and 1 lb of fertilizer. A package of potting mixture requires 1 lb of soil, 2 lb of peat moss, and 3 lb of fertilizer. She has at most 16 lb of soil, 11 lb of peat moss, and 15 lb of fertilizer. A package of garden mixture sells for \$3 and a package of potting mixture sells for \$5.

a) (8 points) Name and define variables. Write an objective function and all necessary constraints:

let x = # of packs of gardening mix let y = # of packs of potting mix

Maximize $z = 3x + 5y$

subject to

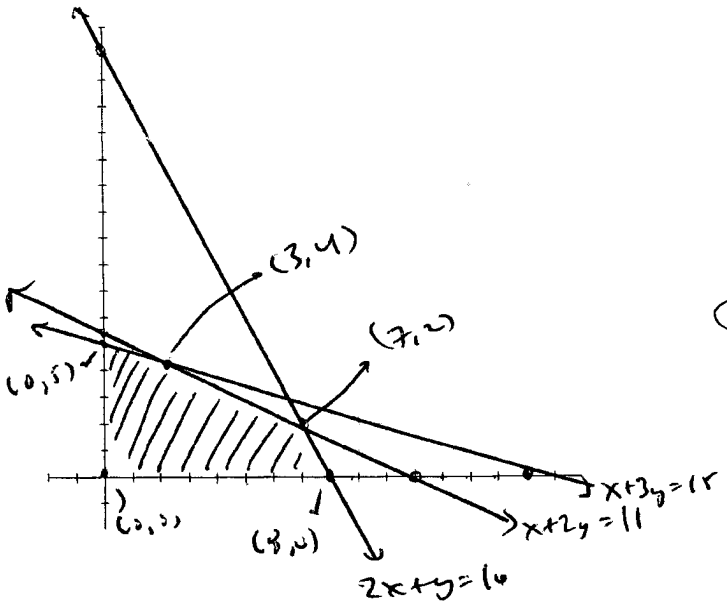
$2x + y \le 16$ soil

$x + 2y \le 11$ peat

$x + 3y \le 15$ fert

$x, y \ge 0$

b) (18 points) Use the Corner Point Method to solve the LP. Interpret the optimal solution and optimal value using the language of the problem:



(x,y)	z = 3x + 5y
(0,0)	0
(0,5)	25
(8,0)	24
(3,4)	29
(7,2)	31

She should make 7 packs of gardening mix & 2 packs of potting mix for a max revenue of \$31.

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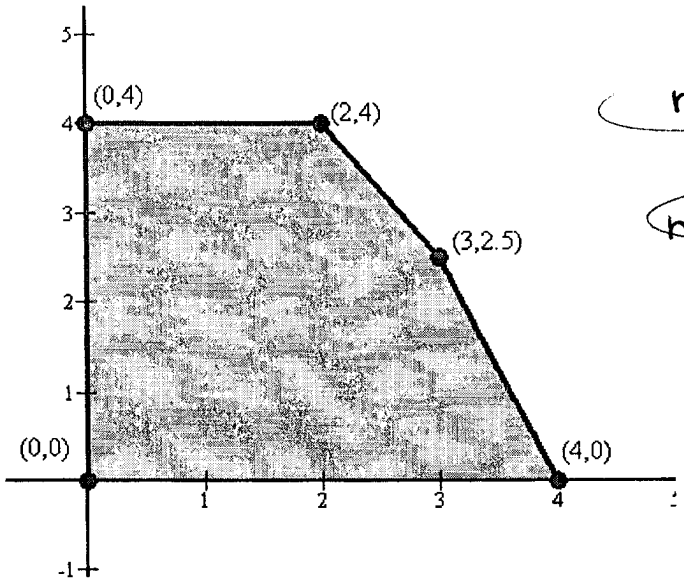
3) (5 points each) For the following feasible regions, find the maximum and minimum of the objective function $z = 2x + 3y$ if they exist. **Be sure to label your answers as "maximum" and minimum".**

a)

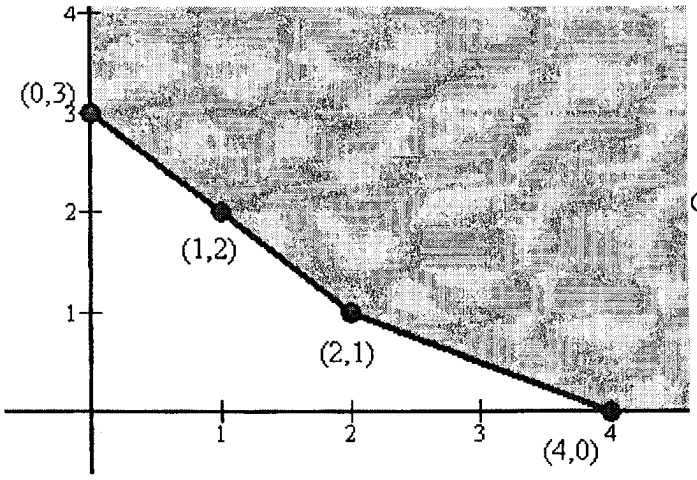


(x, y)	$z = 2x+3y$
(0,0)	6
(0,4)	12
(2,4)	16
(3,2.5)	13.5
(4,0)	8

min → (0,0)

MAX → (2,4)

b)



(x, y)	$z = 2x+3y$
(0,3)	9
(1,2)	8
(2,1)	7
(4,0)	8

min (2,1)

NO MAX!

10

4) (3 points each) Consider a Maximization LP in standard form…

a) How is Pivot Column determined?

che fico!

b) Based on your answer from part *a*, why is the Pivot Column picked this way?

non lo so.

c) Graphically, what does the Pivot Column tell you?

allora

d) How is the Pivot Row determined?

che fai oggi?

e) Based on your answer to part *d*, why is the Pivot Row picked this way?

buona giornata!

f) Graphically, what does the Pivot Row tell you?

I miei pantaloni sono infiammati

5) (6 points each) The following tableaus need pivoting. Circle the pivot element or explain why there isn't one. **Do not actually pivot.** Also state what the current value of z is and the corresponding augmented coordinates:

a)

BV	x_1	x_2	s_1	s_2	z	RHS	
s_1	1	3	1	0	0	100	100/3
s_2	4	2	0	1	0	500	500/2
z	−1	−2	0	0	1	0	

(0, 0, 100, 500) $z = 0$

b)

BV	x_1	x_2	x_3	s_1	s_2	z	RHS
s_1	1	0	−6	1	1	0	300
x_2	3	1	0	0	2	0	50
z	1	2	−5	0	8	1	0

No pivot element. pivot element must be a positive number.

~~(0, 0, 0, 300, 3)~~ $z = 0$

(0, 50, 0, 300, 0) 30

6) For the following Minimization LP…

Minimize $w = 16y_1 + 11y_2 + 15y_3$

subject to

$$2y_1 + y_2 + y_3 \geq 3$$
$$y_1 + 2y_2 + 3y_3 \geq 5$$
$$y_1, y_2, y_3 \geq 0$$

$$\begin{bmatrix} 2 & 1 & 1 & 3 \\ 1 & 2 & 3 & 5 \\ 16 & 11 & 15 & 0 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 & 16 \\ 1 & 2 & 11 \\ 1 & 3 & 15 \\ 3 & 5 & 0 \end{bmatrix}$$

a) (8 points) Write the dual Maximization LP:

Maximize $z = 3x_1 + 5x_2$

subject to

$$2x_1 + x_2 \leq 16$$
$$x_1 + 2x_2 \leq 11$$
$$x_1 + 3x_2 \leq 15$$
$$x_1, x_2 \geq 0$$

b) ~~(12~~ points) Use the Simplex Method on the LP found from part *a*:

BV	x_1	x_2	s_1	s_2	s_3	z	RHS
s_1	2	1	1	0	0	0	16
s_2	1	2	0	1	0	0	11
s_3	1	3	0	0	1	0	15
z	−3	−5	0	0	0	1	0

$\frac{1}{3}R_3 \to R_3$, $-R_3 + R_1 \to R_1$, $-2R_3 + R_2 \to R_2$, $5R_3 + R_4 \to R_4$

BV	x_1	x_2	s_1	s_2	s_3	z	RHS
s_1	5/3	0	1	0	−1/3	0	11
s_2	1/3	0	0	1	−2/3	0	1
x_2	1/3	1	0	0	1/3	0	5
z	−4/3	0	0	0	5/3	1	25

$3R_2 \to R_2$, $-5/3\,R_2 + R_1 \to R_1$, $-1/3\,R_2 + R_3 \to R_3$, $4/3\,R_2 + R_4 \to R_4$

BV	x_1	x_2	s_1	s_2	s_3	z	RHS
s_1	0	0	1	−5	3	0	6
x_1	1	0	0	3	−2	0	3
x_2	0	1	0	−1	1	0	4
z	0	0	0	4	−1	1	29

$\frac{1}{3}R_1 \to R_1$, $2R_1 + R_2 \to R_2$, $-R_1 + R_3 \to R_3$, $R_1 + R_4 \to R_4$

BV	x_1	x_2	s_1	s_2	s_3	z	RHS
s_3	0	0	1/3	−5/3	1	0	2
x_1	1	0	2/3	−1/3	0	0	7
x_2	0	1	−1/3	2/3	0	0	2
z	0	0	1/3	7/3	0	1	31

c) (8 points) What is the optimal solution and optimal value of the Minimization LP?

$w = 31 \quad y_1 = 1/3 \quad y_2 = 7/3 \quad y_3 = 0$

Extra Credit: What is the optimal solution and optimal value of the Maximization LP?

$z = 31 \quad x_1 = 7 \quad x_2 = 2$

so… familiar…