

1) (3 points each) Find the equation of the line in slope-intercept form that...

a) Passes through the points (5,8) and (12,6):

$$m = \frac{6-8}{12-5} = -\frac{2}{7}$$

$$y - 8 = -\frac{2}{7}(x-5)$$
$$y = -\frac{2}{7}x + \frac{66}{7}$$

b) Is through the point (-1,3) and is perpendicular to the line $2x+3y=15$:

$$2x+3y=15 \Rightarrow y = -\frac{2}{3}x+5 \quad m = -\frac{2}{3} \quad \perp m = \frac{3}{2}$$

$$y-3 = \frac{3}{2}(x+1) \Rightarrow y = \frac{3}{2}x + \frac{9}{2}$$

2) (4 points each) Suppose that the price and demand for a gallon of Tuscan Milk was given by $p = D(q) = 74 - 0.26q$ and the price and supply was given by $p = S(q) = 0.48q$ where p is price in dollars and q is the demand in hundreds of ~~three-wheeler~~ *gallons of Tuscan Milk*

a) Find and interpret, using the language of the problem, the following:

i) $D(45) = 74 - 0.26(45)$
 $= 62.3$

4500 gallons are demanded when the price is \$62.30

ii) $S(105) = 0.48(105) = 50.4$

10500 gallons are supplied when the price is \$50.40.

b) Find the demand when the price is \$61:

$$61 = 74 - 0.26q$$

$$\Rightarrow q = 50$$

c) Find the supply when the price is \$60:

$$60 = 0.48q \Rightarrow q = 125$$

d) Find the equilibrium quantity and equilibrium price:

$$D(q) = S(q)$$

$$74 - 0.26q = 0.48q$$

$$74 = 0.74q \Rightarrow q = 100$$

$$p = S(100) = 0.48(100) = 48$$

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3) (2 points each) Short answer: In your own words, describe the formula for...

a) Cost

b) Revenue

c) Profit

Cost

Revenue

Profit

4) (4 points each) Andoio makes the new Snozzberry Pi computer and sells them for \$66.50 each. The cost to build each computer is \$24.50 and there is an additional cost of \$2100 for tools and various supplies.

a) Write and label the corresponding Revenue, Cost, and Profit functions for this problem:

$$R(x) = 66.50x$$

$$C(x) = 24.50x + 2100$$

$$P(x) = R(x) - C(x) = 66.50x - (24.50x + 2100) = 42x - 2100$$

b) When only 40 computers are sold, is there a profit or a loss? Show your work to support your answer.

$$P(40) = 42(40) - 2100$$

$$= -420$$

Loss of \$420.

c) How many computers must be sold to break even?

$$R(x) = C(x)$$

$$66.50x = 24.50x + 2100$$

$$42x = 2100$$

$$x = 50 \text{ computers}$$

5) (3 points each) Consider the following data (source: Census.gov):

Year	2010	2011	2012	2013	2014	2015	2016
Percentage of people 25 years or older that have completed 4 years of college or more	29.9	30.4	30.9	31.7	32.0	32.5	33.4

Let x be the number of years since 2010 and let y be the percentage of people 25 years or older that have completed 4 years of college or more.

a) Find the equation of the regression line. Round values to two decimal places:

$$y = 0.56x + 29.85$$

b) Interpret the slope and y-intercept using the language of the problem:

words go here!

c) Predict the percentage of people that completed more than 4 years in the year 2017:

$$2017 - 2010 = 7$$

$$y = 0.56(7) + 29.85$$

$$= \boxed{33.77\%}$$

d) When will 40% of people 25 or older have completed 4 or more years of college?

$$40 = 0.56x + 29.85$$

$$\Rightarrow x = 18.125$$

+ 2010

During 2028

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6) (8 points each) Solve the system $\begin{cases} 4x - 2y = 14 \\ x + 5y = -13 \end{cases}$ using the methods listed below. Write answer as an ordered ~~trip~~ pair

a) The Echelon (Elimination) method:

$$\begin{array}{r} 4x - 2y = 14 \\ -4x - 20y = 52 \\ \hline -22y = 66 \\ y = -3 \end{array}$$

$$\begin{aligned} x + 5(-3) &= -13 \\ x &= 2 \end{aligned}$$

$$(2, -3)$$

b) Gauss-Jordan method:

$$\begin{aligned} &\begin{bmatrix} 4 & -2 & | & 14 \\ 1 & 5 & | & -13 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 5 & | & -13 \\ 4 & -2 & | & 14 \end{bmatrix} \\ &\xrightarrow{-4R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 5 & | & -13 \\ 0 & -22 & | & 66 \end{bmatrix} \xrightarrow{\begin{array}{l} -\frac{1}{22}R_2 \rightarrow R_2 \\ -5R_2 + R_1 \rightarrow R_1 \end{array}} \begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & -3 \end{bmatrix} \end{aligned}$$

$$(2, -3)$$

7) (1 point) Verify that you made absolutely sure that your answer to 6a is the same as in 6b by signing your name here Auron. You will not receive the credit if the work does not support the same answer.

8) For the following problem:

A person invested \$4,200 for one year, part at 8%, part at 10%, and the remainder at 12%. The total annual return was \$716. The total amount of money invested in the 12% was \$300 more than the amounts invested at 8% and 10% combined. How much was invested at each rate?

a) (2 points) Name and define your variables for this problem:

$x =$ amount invested at 8%

$y =$ " " " 10%

$z =$ " " " 12%

b) (3 points) Set up BUT DO NOT SOLVE a system of equations for this problem:

$$\begin{cases} x + y + z = 4200 \\ .08x + .10y + .12z = 716 \\ z = 300 + x + y \quad \text{or} \quad -x - y + z = 300 \end{cases}$$

9) (3 points each) For the following matrices:

$$A = \begin{bmatrix} 5 & -2 \\ 9 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & -2 & 4 \\ 1 & 12 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 6 & 1 \\ -4 & -2 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 8 \\ 5 & 12 \\ 3 & 4 \end{bmatrix}$$

Find the following or explain why they do not exist:

a) $4A - 5C$

$$\begin{bmatrix} 20 & -8 \\ 36 & 0 \end{bmatrix} + \begin{bmatrix} -30 & -5 \\ 20 & 10 \end{bmatrix}$$

$$\begin{bmatrix} -10 & -13 \\ 56 & 10 \end{bmatrix}$$

b) $6B + D$

\emptyset
not the same dimension

c) BD

$$\begin{bmatrix} 6 & -2 & 4 \\ 1 & 12 & 1 \end{bmatrix} \begin{bmatrix} -2 & 8 \\ 5 & 12 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} -10 & 40 \\ 61 & 156 \end{bmatrix}$$

10) (2 points each) What property must be true to...

a) Add or subtract matrices?

b) Multiply matrices?

both must
be
real matrices

use lancet

11) (6 points part a; 3 points part b) For the system
$$\begin{cases} 3x - y = 2 \\ x - 2y + 2z = -2 \dots \\ 2x - 3y + 3z = -1 \end{cases}$$

a) Find the inverse of the coefficient matrix algebraically using the Gauss-Jordan Method:

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 0 & 1 & 0 & 0 \\ 1 & -2 & 2 & 0 & 1 & 0 \\ 2 & -3 & 3 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_1 \leftrightarrow R_2 \\ -3R_1 + R_2 \rightarrow R_2 \\ -2R_1 + R_3 \rightarrow R_3}} \left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -6 & 1 & -3 & 0 \\ 0 & 1 & -1 & 0 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{\substack{R_2 \leftrightarrow R_3 \\ 2R_2 + R_1 \rightarrow R_1 \\ -5R_2 + R_3 \rightarrow R_3}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & -1 & 0 & -2 & 1 \\ 0 & 0 & -1 & 1 & 7 & 5 \end{array} \right] \xrightarrow{\substack{-R_3 \rightarrow R_3 \\ R_3 + R_2 \rightarrow R_2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & -1 & -9 & 6 \\ 0 & 0 & 1 & -1 & -7 & 5 \end{array} \right]$$

b) Solve the system using the matrix inverse from part a:

$$\begin{bmatrix} 0 & -3 & 2 \\ -1 & -9 & 6 \\ -1 & -7 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \\ 7 \end{bmatrix} \quad (4, 10, 7)$$

x y z

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