Example 14 Suppose you have a six-sided die where the faces for numbers 1, 2 and 3 are red, and the faces for numbers 4, 5 and 6 are green. If you roll the die one time, what is the probability of getting an odd number given that the face is red?

Solution Let *O* be the event of an odd number and *R* be the event of getting a red face. Since the faces for 1 and 3 are red, we know that $P(O \text{ AND } R) = \frac{2}{6}$. Using the formula for conditional probability yields

$$P(R \text{ AND } O) = P(R) \cdot P(O \mid R)$$

$$\frac{2}{6} = \frac{3}{6} \cdot P(O \mid R)$$

$$\frac{1}{3} = \frac{1}{2} \cdot P(O \mid R)$$

We need to solve for $P(O \mid R)$, the probability of an odd number given a red face. To finish off the problem, multiply the last equation by 2 to get

$$P(O \mid R) = \frac{2}{3} = 0.\overline{6}$$

Example 15 At a certain factor there are 150 workers. Everyone has a specialty, but some people have more than one specialty. There are 70 electrical specialists and 100 chemical specialists. Additionally, 10 people are both electrical and chemical specialists. If one person is selected at random from this factory, determine the following probabilities.

a) What is the probability that the person will be an electrical specialist, given that person is a chemical specialist?

b) What is the probability that the person will be a chemical specialist, given that person is an electrical specialist.

Solutions Let *E* be the event of an electrical specialist, *M* be the event of a mechanical specialist and *C* the event of a chemical specialist.

a) We need to determine $P(E \mid C)$. Use the conditional probability formula.

$$P(C \text{ AND } E) = P(C) \cdot P(E \mid C)$$

Plugging in the probabilities yields

$$P(C \text{ AND } E) = P(C) \cdot P(E \mid C)$$

$$\frac{10}{150} = \frac{100}{150} \cdot P(E \mid C)$$

$$\frac{1}{15} = \frac{2}{3} \cdot P(E \mid C)$$

$$\frac{3}{2} \cdot \frac{1}{15} = P(E \mid C)$$

Thus,

$$P(E \mid C) = \frac{3}{30} = \frac{1}{10} = 0.1$$

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b) We need to determine $P(C \mid E)$. Once again, we go to the conditional probability equation.

$$P(E \text{ AND} C) = P(E) \cdot P(C \mid E)$$

$$\frac{10}{150} = \frac{70}{150} \cdot P(C \mid E)$$

$$\frac{1}{15} = \frac{7}{15} \cdot P(C \mid E)$$

Multiply both sides of the equation by $\frac{15}{7}$ to get

$$P(C \mid E) = \frac{1}{7} \approx 0.143$$

Example 16 A mayoral election is held in a small city. There are three candidates running for mayor. The results are broken down according to the gender of the voters. The following table contains the results.

Gender	Democrat	Republican	Other	Totals
Males	538	694	30	1262
Females	621	740	9	1370
Totals	1159	1434	39	2632

If one voter is selected at random, then determine the following probabilities.

a) What is the probability that the person voted republican, given that person was female?

b) What is the probability that the person was male, given that they voted other?

Solutions Let *F, M, D, R, O* stand for the obvious events.

a) We need to determine $P(R \mid F)$. Use the conditional probability formula.

$$P(R \text{ AND } F) = P(F) \cdot P(R \mid F)$$

$$\frac{740}{2632} = \frac{1370}{2632} \cdot P(R \mid F)$$

$$\frac{2632}{1370} \cdot \frac{740}{2632} = P(R \mid F)$$

$$\frac{740}{1370} = P(R \mid F)$$

$$P(R \mid F) \approx 0.540$$

b) We need to determine $P(M \mid O)$. Use the conditional probability formula.

$$P(O \text{ AND } M) = P(O) \cdot P(M \mid O)$$

$$\frac{30}{2632} = \frac{39}{2632} \cdot P(M \mid O)$$

$$\frac{2632}{39} \cdot \frac{30}{2632} = P(M \mid O)$$

$$P(M \mid O) = \frac{30}{39} \approx 0.769$$

Note For events *A* and *B*, we have

$$P(A \text{ AND } B) = P(B \text{ AND } A)$$

You'll notice in the examples that sometimes it's easier to put *A* first, and *B* second, and sometimes vice versa is preferred. This is not necessary, but I find it easier to remember the formula for conditional probability.

Exercises

Problems 25 - 30 involve selecting one card, at random, from a deck of 52 cards. There are 26 red cards, 16 face cards, and 8 red face cards. Additionally, there are 4 jacks, of which 2 are red. Note that jacks are face cards.

25. What is the probability of getting a face card, given that the card is red?

26. What is the probability of getting a red card, given that it is a face card?

27. What is the probability of getting a red card, given that the card is a jack?

28. What is the probability of getting a jack, given that the card is red?

29. What is the probability of getting a jack, given that the card is a face card?

30. What is the probability of getting a face card, given that it is a jack?

Problems 31 - 36 concern the following scenario. A conference has 735 psychiatrists in attendance. 490 of these psychiatrists believe in the Freudian method of psychoanalysis, 510 believe in the Jungian method of psychoanalysis, and 380 believe in the Sullivan method of psychoanalysis. Additionally, 215 of these psychiatrists believe in Freud and Jung's methods, 110 believe in both Freud and Sullivan's methods, and 98 believe in both Jung and Sullivan's methods. If one of these psychiatrists is selected at random, determine the following probabilities.

31. What is the probability of a Freudian, given a Jungian?

32. What is the probability of a Jungian, given a Freudian?

33. What is the probability of a Freudian, given a Frommian?

34. What is the probability of a Frommian, given a Jungian?

35. What is the probability of a Freudian, given that the psychiatrist is non-Jungian?

36. What is the probability of a Jungian, given that the psychiatrist is non-Freudian?

Problem 37 - 42 the handedness of bowling balls found in a bowling alley. A certain, progressive bowling alley contains bowling balls for right-handers, left-handers, and even neutral (meaning they are neither right nor left-handed). Additionally, the bowling balls are either orange, black or an orange-black swirl. The following table lists the numbers.

Handedness	**Orange**	**Black**	**Swirl**	**Totals**
Left	85	95	70	250
Right	220	250	180	650
Neither	45	45	10	100
Totals	350	390	260	1000

If you were to choose one bowling ball, at random, from these bowling balls, determine the following probabilities.

37. What is the probability that the ball is black, given that it is for a left-hander?

38. What is the probability that the ball is for a left-hander, given that it is black?

39. What is the probability that the ball is for neither left or right-hander, given that it is swirled?

40. What is the probability that the ball is swirled, given that it is neither for a left or right-hander?

41. What is the probability that the ball is for a right-hander, given that it is orange?

42. What is the probability that the ball is orange, given that it is for a right-hander?