1) (2 points) Solve for the variable in $x^4 + 9x^2 + 20 = 0$:

$(x^2 + 5)(x^2 + 4) = 0$

$x^2 = -5 \Rightarrow \boxed{x = \pm\sqrt{5}\, i}$ $\quad x^2 = -4 \Rightarrow \boxed{x = \pm 2i}$

2) (2 points each) For the function $f(x) = 2x^2 - 8x + 1$, determine…

a) If it opens up or down. How do you know?

up $a = 2 > 0$

b) The coordinates of the vertex: $(2, -7)$

$x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = 2$

$f(2) = 2(2)^2 - 8(2) + 1 = -7$

c) The domain: $\mathbb{R}$

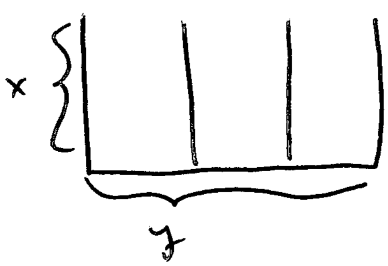
d) The range: $[-7, \infty)$

e) Interval of increase: $(2, \infty)$

f) Interval of decrease: $(-\infty, 2)$

3) (5 points) Dr. Mort, an expert in crow migration, is suddenly upset with the new crop of teaching assistants he has this year. He decides to enclose his assistants in an evil rectangular garden of doom next to his laboratory. He wants to create 3 adjacent pens that all use his laboratory as an edge while only using 250 feet of fence. The edge against the laboratory will not need fencing. What should the dimensions of the enclosure be to maximize area? Also, what is the maximum area?

Dr. Mort, seen here with his favorite teaching assistant, Gary.



$4x + y = 250 \Rightarrow y = 250 - 4x$

$A = xy$

$A(x) = x(250 - 4x) = -4x^2 + 250x$

$\boxed{x = \frac{-b}{2a} = \frac{-250}{2(-4)} = 31.25 \text{ ft} \quad y = 250 - 4(31.25) = 125 \text{ ft}}$

$\boxed{A = xy = 3906.25 \text{ ft}^2}$

4) (2 points each) Solve for the variable. Write part *b* in interval notation:

a) $\frac{3x-7}{x^2-9} - \frac{5}{x-3} = \frac{7}{x+3}$

multiply by $(x+3)(x-3)$

$3x - 7 - 5(x+3) = 7(x-3)$

$3x - 7 - 5x - 15 = 7x - 21$

$-1 = 9x$

$\boxed{x = -1/9}$

b) $3|x-7| - 5 \geq 8$

$|x-7| \geq \frac{13}{3}$

$x - 7 \geq \frac{13}{3}$ or $x - 7 \leq -\frac{13}{3}$

$x \geq \frac{34}{3}$ or $x \leq \frac{8}{3}$

$\frac{8}{3}$ $\quad \frac{34}{3}$

$(-\infty, \frac{8}{3}] \cup [\frac{34}{3}, \infty)$

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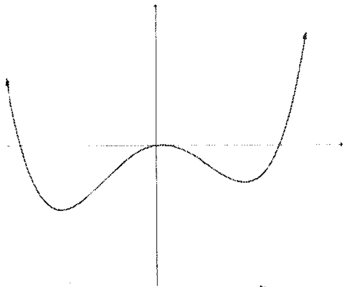
5) (2 points each) Give an example of a function which will have similar arrowheads to the function below:

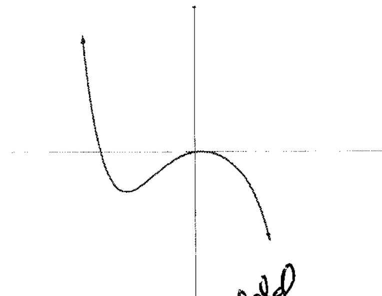
a)



$f(x) =$ (positive)$(x)^{\text{even}}$

? lead term

b)



$f(x) =$ (negative)$(x)^{\text{odd}}$

? leading term

6) (3 points each) For the function $f(x) = (x+4)^2(2x+1)^2(x-3)\ldots$

a) Find the leading term and state which quadrants the arrowheads will be in:

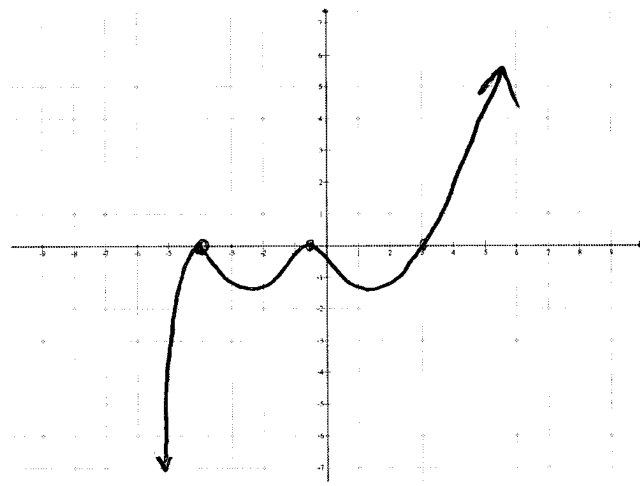
Lead term: $4x^5$

Q: I & III

b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
-4	2	Touch
$-1/2$	2	Touch
3	1	Cross

c) Sketch the graph based on parts *a* and *b*:



7) (3 points each) For the function $f(x) = -3x^4 + 12x^2\ldots = -3x^2(x^2-4)$ → $(x+2)(x-2)$

a) Find the leading term and state which quadrants the arrowheads will be in:

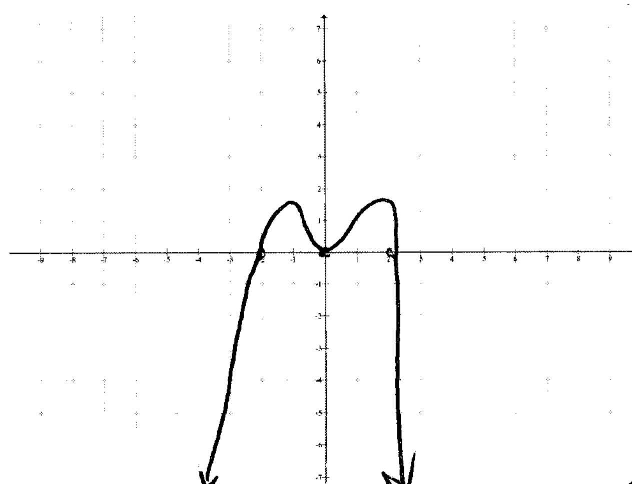
LT: $-3x^4$

Q: III & IV

b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
0	2	Touch
2	1	Cross
-2	1	Cross

c) Sketch the graph based on parts *a* and *b*:



TZ

8) (2 points each) Form a polynomial function of degree four that meets the following requirements. **Be sure to leave your answer in factored form**:

a) Has zeros at 8, 3, 5, and 0:

$$f(x) = (x-8)(x-3)(x-5)x$$

b) Has the same zeros and multiplicity as in part *a* but is a different function:

$$g(x) = \underbrace{2\pi e^2}_{\text{any nonzero \#}}(x-8)(x-3)(x-5)x$$

c) Has a zero at $2+3i$, and 8 is a zero of multiplicity 2:

$$h(x) = (x-(2+3i))(x-(2-3i))(x-8)^2$$

9) (3 pts *a;* 2 pts others) Consider the functions $f(x) = 6x^3 + x^2 - 12x + 5$ and $g(x) = 3x^2 + 2x - 5$.

a) Divide $f(x)$ by $g(x)$ using long division:

$$
\begin{array}{r l}
 & 2x - 1 \\
3x^2+2x-5 \,\big) & 6x^3 + x^2 - 12x + 5 \\
 & -(6x^3 + 4x^2 - 10x) \\
\hline
 & -3x^2 - 2x + 5 \\
 & -(-3x^2 - 2x + 5) \\
\hline
 & 0
\end{array}
$$

b) Based on your work in part *a*, was $g(x)$ a factor of $f(x)$? Why or why not?

Yup!

the remainder was zero.

c) What is the equation of the oblique asymptote of the rational function $y = \dfrac{6x^3 + x^2 - 12x + 5}{3x^2 + 2x - 5}$?

$$y = 2x - 1$$

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10) (7 points each) Factor the polynomial completely by first listing the possible rational roots and then using synthetic division and your calculator.

a) $f(x) = x^3 - 5x^2 - 12x + 36$

$p: \pm 1, 2, 3, 4, 6, 9, 12, 18, 36$

$q: \pm 1;\ \frac{p}{q} = \pm 1, 2, 3, 4, 6, 9, 12, 18, 36$

$$\begin{array}{r|rrrr} -3 & 1 & -5 & -12 & 36 \\ & & -3 & 24 & -36 \\ \hline 2 & 1 & -8 & 12 & 0 \\ & & 2 & -12 & \\ \hline & 1 & -6 & 0 & \end{array}$$

$\boxed{f(x) = (x+3)(x-2)(x-6)}$

b) $g(x) = x^4 - 2x^3 - 10x^2 + 16x + 40$

$p = \pm 1, 2, 4, 5, 8, 10, 20, 40$

$q = \pm 1;\ \frac{p}{q} = \pm 1, 2, 4, 5, 8, 10, 20, 40$

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -10 & 16 & 40 \\ & & -2 & 8 & 4 & -40 \\ \hline -2 & 1 & -4 & -2 & 20 & 0 \\ & & -2 & 12 & -20 & \\ \hline & 1 & -6 & 10 & 0 & \end{array}$$

$g(x) = (x+2)^2(x^2 - 6x + 10)$ quadratic formula

$= (x+2)^2(x-(3+2i))(x-(3-2i))$

11) (4 points each) For the function $f(x) = \dfrac{x^2+6x+9}{x^3+7x^2+12x}$, find…

a) The domain:

$x^3 + 7x^2 + 12x = 0$

$x(x+3)(x+4) = 0$

$\boxed{x \neq 0, -3, -4}$

b) The x- and y-intercepts:

x-int:

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0 \Rightarrow x = -3$ Not in domain $\emptyset$

y-int:

$f(0) = \frac{9}{0}$ $\emptyset$ not in range $\emptyset$

No intercepts

c) Any vertical asymptotes and holes:

$x = 0$: $0^2 + 6(0) + 9 \neq 0$ $\boxed{x = 0 \text{ VA}}$

$x = -4$: $(-4)^2 + 6(-4) + 9 \neq 0$ $\boxed{x = -4 \text{ VA}}$

$x = -3$: $(-3)^2 + 6(-3) + 9 = 0$ Hole!

$\dfrac{(x+3)^2}{x(x+3)(x+4)} = \dfrac{x+3}{x(x+4)}$ @ $x = -3$ $\dfrac{-3+3}{-3(-3+4)} = 0$

Hole at $(-3, 0)$

d) Any horizontal or oblique asymptotes:

HA $y = 0$

degree of num < degree of denom.

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12) (2 points each) Fill in the blank:

a) If c is a zero of a function f, then $f(c) =$ tonight, and I'm thinking is a factor.

b) Numbers not in the domain of a rational function lead to about eating Chinese food.

13) (3 points each) Solve for the variable. Write answer in interval notation:

a) $x^3 - 5x^2 + 6x \le 0$

① $x(x-3)(x-2) = 0$

$x = 0, 3, 2$

② ③ $-\quad +\quad -\quad +$

$0 \quad 2 \quad 3$

④ $\boxed{(-\infty, 0] \cup [2, 3]}$

b) $\dfrac{x^2 + 6x + 5}{x - 4} \ge 0$

① $x^2 + 6x + 5 = 0 \Rightarrow x = -5, -1$

② $x - 4 = 0 \Rightarrow x = 4$

③ ④ $-\quad +\quad -\quad +$

$-5 \quad -1 \quad 4$

⑤ $[-5, -1] \cup (4, \infty)$

? do not include!

It causes division by zero.

Extra Credit (2 points) :

Find the equation of a rational function in factored form that has the following properties:

a) Hole at $x = 9$

b) Vertical Asymptotes at $x = -3$ and $x = 5$

c) x-intercepts at $x = \frac{1}{7}$ and $x = -6$

d) Horizontal asymptote at $y = 7$

$$f(x) = \frac{\overset{a}{(x-9)}\overset{c+d}{(7x-1)}\overset{c}{(x+6)}}{\underset{a}{(x-9)}\underset{b}{(x+3)}\underset{b}{(x-5)}}$$

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