1) (6 points) Solve for the triangle. Be sure to show all necessary work:

LoS

$A = 42^\circ$ $\quad a = 15$

$B = 57^\circ$ $\quad b = \underline{19}$

$C = \underline{81^\circ}$ $\quad c = \underline{22}$

$180 - (42 + 57^\circ)$

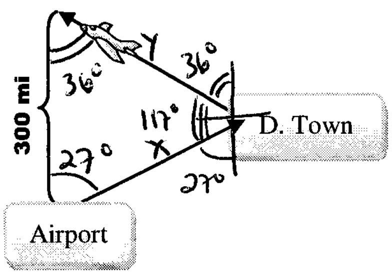
$\frac{b}{\sin 57^\circ} = \frac{15}{\sin 42^\circ} \Rightarrow b = \frac{15 \sin 57^\circ}{\sin 42^\circ} \approx 19$

$\frac{c}{\sin 81^\circ} = \frac{15}{\sin 42^\circ} \Rightarrow c = \frac{15 \sin 81^\circ}{\sin 42^\circ} \approx 22$

2) (6 points) Math Airways drops calculators to students in need. A plane leaves the airport bearing $N27^\circ E$ towards Division Town. While flying over Division Town, the plane heads on a bearing of $N36^\circ W$. After some time, it is 300 miles due north of the airport. See picture:

a) Find the total number of miles the plane flew to and from Division Town. Round only at the end:

LoS




$x = \frac{300 \sin 36^\circ}{\sin 117^\circ} \approx 198 \text{ mi}$

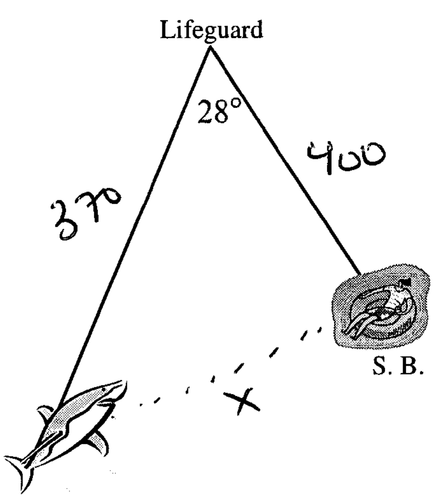
$y = \frac{300 \sin 27^\circ}{\sin 117^\circ} \approx 153 \text{ mi}$

$\boxed{351 \text{ mi}}$

b) (2 points) Extra Credit: If the plane dropped one calculator every 5 feet, how many calculators were dropped on this trip?

$351 \text{ mi} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ calc}}{5 \text{ ft}} \approx 370{,}656$ calculators

3) (4 points) Silly Bill floats in the water 400 ft from a lifeguard. The lifeguard, turning 28° spots a shark 370 ft away. How far is the shark from Silly Billy?




LoC

$x = \sqrt{370^2 + 400^2 - 2 \cdot 370 \cdot 400 \cdot \cos 28^\circ}$

$x \approx 189 \text{ ft}$

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4) (2 points) Concerning the given information of a triangle, how do you know when to use the Law of Sines versus the Law of Cosines?

5) (3 points) Find the trigonometric form of the complex number $5+5\sqrt{3}i$:

$r = \sqrt{5^2+(5\sqrt{3})^2} = \sqrt{100} = 10$

$\cos\theta = \frac{5}{10} = \frac{1}{2}$
$\sin\theta = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$ $\Big\}$ $\theta = 60°$ or $\pi/3$

$\boxed{10(\cos 60° + i\sin 60°)}$

6) (3 points) Find the standard form of the complex number $\sqrt{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$= \sqrt{2}\left(\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i}$

7) (4 points) For the complex numbers $z_1 = \frac{\sqrt{2}}{2}+\frac{\sqrt{6}}{2}i$ and $z_2 = 5+5\sqrt{3}i$, find $z_1 \times z_2$, using the trigonometric forms and the formula $z_1 \times z_2 = r_1 \times r_2\left[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right]$. Write final answer in standard form:

$z_1 \cdot z_2 = 10 \cdot \sqrt{2}(\cos(60°+60°) + i\sin(60°+60°))$

$= 10\sqrt{2}(\cos 120° + i\sin 120°) = 10\sqrt{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \boxed{-5\sqrt{2} + 5\sqrt{6}i}$

8) (4 points part *a*; 6 points part *b*) For the complex number $-2i = 2(\cos 270° + i\sin 270°)$, find the following. For part *a*, use the formula $(a+bi)^n = r^n\left[\cos(n\theta)+i\sin(n\theta)\right]$. For part *b*, use the formula $(a+bi)^{\frac{1}{n}} = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta}{n}+\frac{360°}{n}\cdot k\right)+i\sin\left(\frac{\theta}{n}+\frac{360°}{n}\cdot k\right)\right]$. Write answers in standard form.

a) $(-2i)^6$

$= 2^6(\cos(6\cdot 270°) + i\sin(6\cdot 270°))$

$= 64(\cos 1620° + i\sin 1620°)$

$= 64(\cos 180° + i\sin 180°)$

$= 64(-1 + 0i)$

$= -64$

b) The cube roots of $-2i$:

$2^{\frac{1}{3}}\left(\cos\left(\frac{270°}{3} + k\cdot\frac{360°}{3}\right) + i\sin\left(\frac{270°}{3} + k\cdot\frac{360°}{3}\right)\right)$

$2^{\frac{1}{3}}(\cos(90° + 120°k) + i\sin(90° + 120°k))$

$k=0$ $2^{1/3}(\cos 90° + i\sin 90°) = \sqrt[3]{2}(0 + 1i) = \boxed{\sqrt[3]{2}i}$

$k=1$ $2^{\frac{1}{3}}(\cos 210° + i\sin 210°) = \sqrt[3]{2}\left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \boxed{-\frac{\sqrt[3]{2}\sqrt{3}}{2} - \frac{\sqrt[3]{2}}{2}i}$

$k=2$ $2^{\frac{1}{3}}(\cos 330° + i\sin 330°) = \sqrt[3]{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$
$= \boxed{\frac{\sqrt[3]{2}\sqrt{3}}{2} - \frac{\sqrt[3]{2}}{2}i}$



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8) (3 points each) For the point $\left(4, \frac{3\pi}{2}\right)$, find a different representation of the point in polar form that satisfies the following conditions:

a) $r < 0$ and $\theta > 0$

$\left(-4, \frac{\pi}{2}\right)$

b) $r > 0$ and $\theta < 0$

$\left(4, -\frac{\pi}{2}\right)$

c) $r > 0$ and $\theta > 0$

$\left(4, \frac{7\pi}{2}\right)$

9) (4 points each) Convert as stated:

a) $\left(\sqrt{2}, -\sqrt{2}\right)$ to polar:

$r = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = 2$

$\left.\begin{array}{l} \cos\theta = \frac{\sqrt{2}}{2} \\ \sin\theta = -\frac{\sqrt{2}}{2} \end{array}\right\} \theta = \frac{7\pi}{4}$

$\left(2, \frac{7\pi}{4}\right)$

b) $\left(9, \frac{11\pi}{6}\right)$ to rectangular:

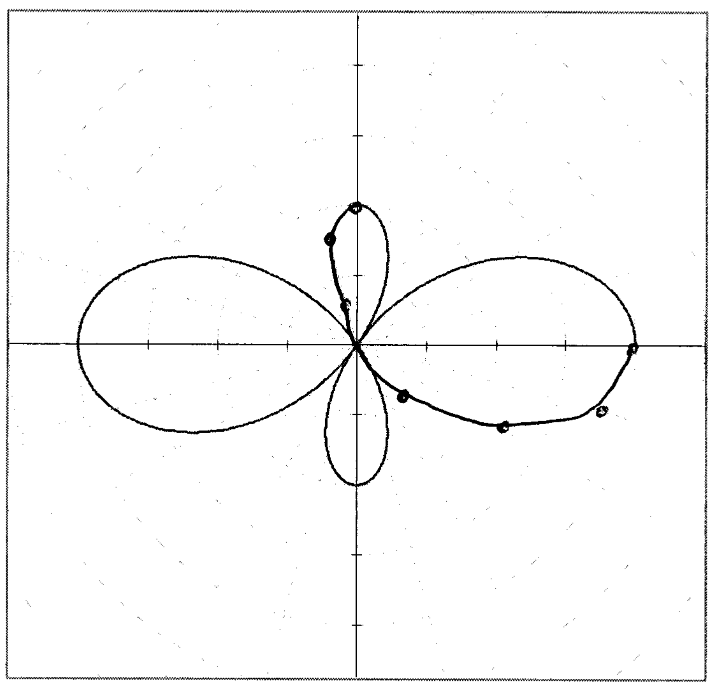
$x = 9\cos\frac{11\pi}{6} = \frac{9\sqrt{3}}{2}$

$y = 9\sin\frac{11\pi}{6} = -\frac{9}{2}$

$\left(\frac{9\sqrt{3}}{2}, -\frac{9}{2}\right)$

10) (6 points) Sketch a graph of $r = 3\cos(2\theta) + 1$. Part of the graph has been done for you. Use the values from $270°$ to $360°$ to finish the graph. Round to one decimal.

| $\theta$ | $r$ |
|---|---|
| 270° | −2 |
| 285° | −1.6 |
| 300° | −0.5 |
| 315° | 1 |
| 330° | 2.5 |
| 345° | 3.6 |
| 360° | 4 |



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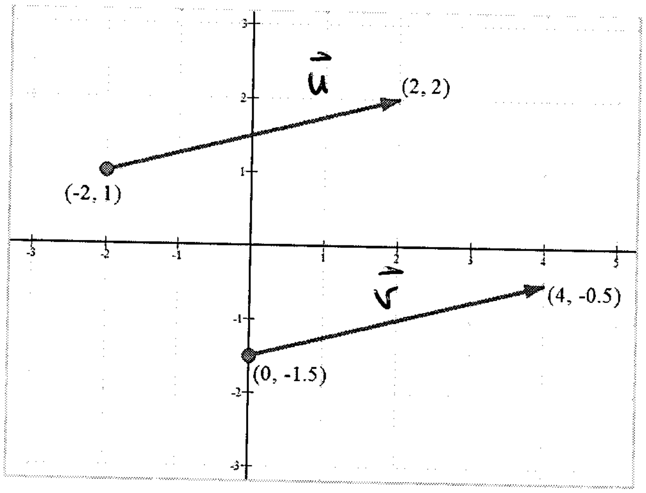
→ leave exact.

11) (5 points) For the given vectors, determine algebraically if they are equivalent:




$m_{\vec{u}} = \frac{2-1}{2-(-2)} = \frac{1}{4}$ $\quad m_{\vec{v}} = \frac{-0.5-(-1.5)}{4-0} = \frac{1}{4}$

$|\vec{u}| = \sqrt{(2-(-2))^2+(2-1)^2} = \sqrt{17}$

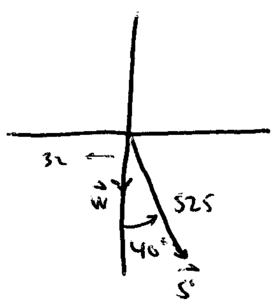
$|\vec{v}| = \sqrt{(4-0)^2+(-0.5-(-1.5))^2} = \sqrt{17}$

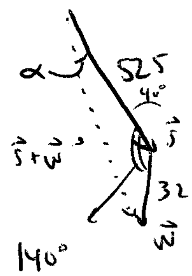
yes $\vec{u} = \vec{v}$

or can say $\vec{u} = \vec{v} = \langle 4, 1 \rangle$

12) (8 points) Balthier, a Dalmascan sky pirate, flies his ship, the Strahl, from a Dalmascan port on a bearing of $S40°E$ at a speed of 525 mph. A wind is blowing from due North at a speed of 32 mph. Find the ground speed and bearing of the ship in the wind. Draw a picture and use the Law of Cosines.

Final Fantasy XII: The Zodiac Age releases in July. I preordered 5 copies.




$|\vec{s}+\vec{w}| = \sqrt{525^2+32^2-2(525)(32)\cos 140°}$

$\approx \boxed{550 \text{ mph}}$

$\alpha = \cos^{-1}\left(\frac{32^2-525^2-550^2}{-2(525)(550)}\right) \approx 2°$ $\quad 40-2=38°$

S38°E

13) (3 points each) Let $\vec{u} = \langle 4,6 \rangle$ and $\vec{v} = \langle 4,19 \rangle$. Find and simplify:

a) $4\vec{u} - \vec{v}$

$\langle 16,24 \rangle - \langle 4,19 \rangle$

$\langle 12, 5 \rangle$

b) $|4\vec{u} - \vec{v}|$

$= \sqrt{12^2+5^2} = 13$

welcome!

c) The unit vector in the same direction as $4\vec{u} - \vec{v}$:

$\left\langle \frac{12}{13}, \frac{5}{13} \right\rangle$

d) $\vec{u} \cdot \vec{v}$:

$= 4 \cdot 4 + 6 \cdot 19$

$= 130$

e) The angle between the vectors $\vec{u}$ and $\vec{v}$. Round to two decimal places:

$\cos\theta = \frac{130}{\sqrt{4^2+6^2} \cdot \sqrt{4^2+19^2}} \Rightarrow \theta = \cos^{-1} \frac{130}{\sqrt{52 \cdot 377}}$

$\theta \approx 21.8°$

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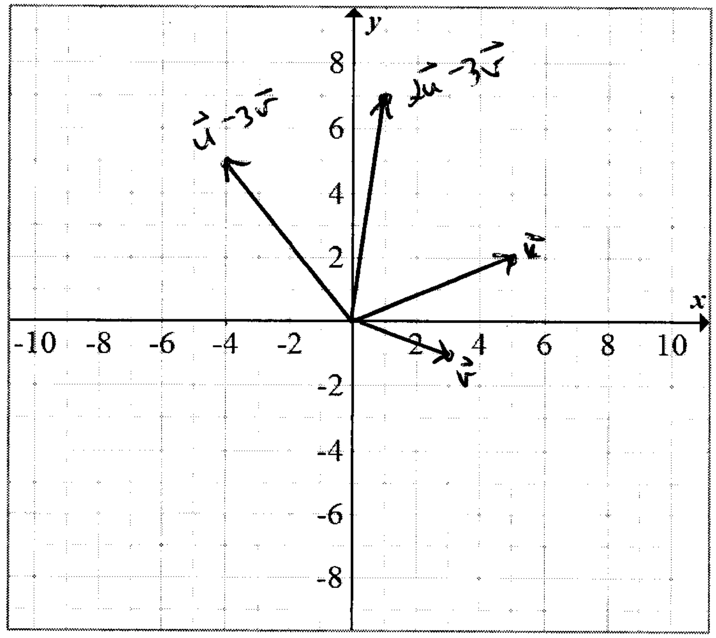
14) (4 points) Given the vector $\vec{u} = \langle 5, 2 \rangle$ and $\vec{v} = \langle 3, -1 \rangle$, draw and label the vectors $\vec{u}, \vec{v}$, $\vec{u} - 3\vec{v}$, and ~~$2\vec{u} - 3\vec{v}$~~ :

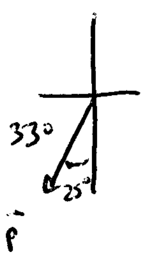
$2\vec{u} - 3\vec{v}$




$\vec{u} - 3\vec{v} = \langle -4, 5 \rangle$

$2\vec{u} - 3\vec{v} = \langle 1, 7 \rangle$

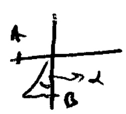
15) (8 points) An airplane travels on a bearing of $S25°W$ with an airspeed of 330 mph. A wind is blowing from the **south** direction at a speed of 40 mph. Find the ground speed and direction of the plane using the formula $\vec{v} = |\vec{v}|(\cos\theta\vec{i} + \sin\theta\vec{j})$. Round only the final answer to two decimal places:




$\vec{p} = 330(\cos 245° \hat{i} + \sin 245° \hat{j})$ $\quad \vec{w} = 40(\cos 90° \hat{i} + \sin 90° \hat{j})$

$\vec{p} + \vec{w} = \underbrace{(330 \cos 245° + 40 \cos 90°)}_{A} \hat{i} + \underbrace{(330 \sin 245° + 40 \sin 90°)}_{B} \hat{j}$


$|\vec{p} + \vec{w}| = \sqrt{A^2 + B^2} \approx \boxed{294 \text{ mph}}$




$\tan \alpha = \frac{A}{B} \Rightarrow \tan^{-1} \frac{A}{B} = \alpha \approx 28°$ $\quad$ S 28° W

16) (5 points) A large, unattended child, pulls a wagon with a force of 15.5 lbs for 800 ft. The handle makes a $52°$ angle to the horizontal. How much work is done by the child in terms of foot-pounds?

$\vec{w} = |\vec{f}||\vec{AB}| \cos \theta$

$\vec{w} = 15.5 \text{lbs} \cdot 800 \text{ft} \cos 52° \approx 7634$ ft-lbs



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