1) (5 points each) Simplify.

a) $\sin x\left(\sin x+\cos^2 x\csc x\right)$

$\sin^2 x + \sin x\cos^2 x\csc x$ cancel

$\sin^2 x + \cos^2 x$

1

b) $-2\sin^3 x\cos x-2\sin x\cos^3 x$

$-\underbrace{2\sin x\cos x}_{\sin 2x}(\underbrace{\sin^2 x+\cos^2 x}_{1})$

$-\sin 2x$

2) (4 points each) Find the exact value of $\cos\frac{\pi}{12}$ using…

a) A Sum or Difference Formula:

$\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) =$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

b) A Half Angle Formula:

$\cos\frac{\pi}{12} = \cos\left(\frac{\pi/6}{2}\right)$

$= \sqrt{\frac{1+\cos\pi/6}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}}$

$= \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

3) (3 points each) Based on either answer in 2a or 2b, determine the exact value of the following. **Do not use formula!**

a) $\cos\frac{11\pi}{12}$ and $\cos\frac{13\pi}{12}$

$-\frac{\sqrt{2}+\sqrt{6}}{4}$

negative

b $\cos\frac{23\pi}{12}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

same

4) (5 points) Simplify the expression: $\dfrac{\sec^4 x-\tan^4 x}{\sec^2 x+\tan^2 x}$

$= \dfrac{\cancel{(\sec^2 x+\tan^2 x)}\overbrace{(\sec^2 x-\tan^2 x)}^{1}}{\cancel{\sec^2 x+\tan^2 x}}$

= one!

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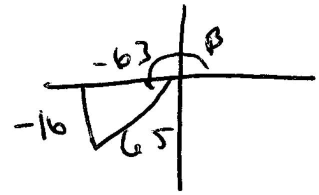


5) (4 points each) Given that $\tan\beta = \dfrac{16}{63}$ and that β is in Quadrant III, find the exact value for…

a) $\sin(2\beta)$: $2\sin\beta\cos\beta$

$2\left(-\frac{16}{65}\right)\left(-\frac{63}{65}\right)$

$= \dfrac{2016}{4225}$

b) $\cos(2\beta)$: $= \cos^2\beta - \sin^2\beta$

$\left(-\frac{63}{65}\right)^2 - \left(-\frac{16}{65}\right)^2$

$\dfrac{3713}{4225}$

c) $\tan(2\beta)$:

$\dfrac{2016}{3713}$

d) The quadrant that 2β resides in:

I

6) (4 points a – c, 6 points 6) Simplify by finding the exact value:

a) $\sin^{-1}\left(\sin\dfrac{\pi}{12}\right)$

$\dfrac{\pi}{12}$

b) $\cos\left(\sin^{-1}\left(-\dfrac{1}{2}\right)\right)$

$\cos\left(-\frac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

c) $\cos^{-1}\left(\cos\left(-\dfrac{5\pi}{4}\right)\right)$

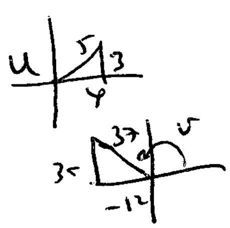
$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$= \dfrac{3\pi}{4}$

d) $\sin\left(\overbrace{\sin^{-1}\dfrac{3}{5}}^{u} + \overbrace{\cos^{-1}\left(-\dfrac{12}{37}\right)}^{v}\right)$

$\sin\left(\sin^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}-\frac{12}{37}\right) + \cos\left(\sin^{-1}\frac{3}{5}\right)\sin\left(\cos^{-1}-\frac{12}{37}\right)$

$\left(\frac{3}{5}\right)\left(-\frac{12}{37}\right) + \left(\frac{4}{5}\right)\left(\frac{35}{37}\right)$



$= \dfrac{104}{185}$

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7) (6 points each) Solve the equation for the variable:

a) $\sqrt{2}\cos x + 1 = 0$:

$\cos x = -\frac{\sqrt{2}}{2}$

$x = \frac{3\pi}{4} + 2\pi k$

$x = \frac{5\pi}{4} + 2\pi k$ $\quad k \in \mathbb{Z}$

b) $\sin\left(\overset{\theta}{2x}\right) = \frac{1}{2}$ on $[0, 2\pi)$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6} + 2\pi k$

$\theta = \frac{5\pi}{6} + 2\pi k$

$2x = \frac{\pi}{6} + 2\pi k$

$2x = \frac{5\pi}{6} + 2\pi k$

$x = \frac{\pi}{12} + \pi k$

$x = \frac{5\pi}{12} + \pi k$

$x \in \left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}\right\}$

8) (6 points) Fill in the blank using interval notation:

	$\sin x$ *	$\cos x$ *	$\tan x$ *	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain						
Range						

just guess one

*Write the domain restrictions for these three functions.

9) Short answer:

a) (4 points) Explain why $\sin\left(\sin^{-1}\frac{1}{2}\right) = \frac{1}{2}$ but $\sin^{-1}\left(\sin\frac{7\pi}{4}\right) \neq \frac{7\pi}{4}$.

boh

b) Extra credit: (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

¿que?

c) Extra credit: (2 points) Referring to number 4, find the exact value for $\sin(4\beta)$:

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