

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

- ❖ Write your name below on the space provided.
- ❖ This test has a total of 6 pages.
- ❖ Work the problem in the space provided. If you need more space, write on the back of the test.
- ❖ To insure maximum credit, show your work. In general, full credit will not be given for unsupported answers.
- ❖ Look only at your test. Don't give me the impression that you are cheating.
- ❖ Be sure to write neatly. If I cannot read what was written, do not expect the problem to be graded.
- ❖ If you finish early, go over the test again.

Good luck!

Number	Maximum	Score
1	10	
2	8	
3	6	
4	5	
5	16	
6	18	
7	12	
8	6	
9	4/8	
10	15	
Total	100	

Name _____

CIRCLE YOUR FINAL ANSWERS!

TEAR OFF THE LAST SHEET!

1) (5 points each) Simplify.

a) $\sin x(\sin x + \cos^2 x \csc x)$

b) $-2\sin^3 x \cos x - 2\sin x \cos^3 x$

2) (4 points each) Find the exact value of $\cos \frac{\pi}{12}$ using...

a) A Sum or Difference Formula:

b) A Half Angle Formula:

3) (3 points each) Based on either answer in 2a or 2b, determine the exact value of the following.

Do not use formula!

a) $\cos \frac{11\pi}{12}$ and $\cos \frac{13\pi}{12}$

b) $\cos \frac{23\pi}{12}$

4) (5 points) Simplify the expression: $\frac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x}$

5) (4 points each) Given that $\tan \beta = \frac{16}{63}$ and that β is in Quadrant III, find the exact value for...

a) $\sin(2\beta)$:

b) $\cos(2\beta)$:

c) $\tan(2\beta)$:

d) The quadrant that 2β resides in:

6) (4 points a – c, 6 points 6) Simplify by finding the exact value:

a) $\sin^{-1}\left(\sin \frac{\pi}{12}\right)$

b) $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

c) $\cos^{-1}\left(\cos\left(-\frac{5\pi}{4}\right)\right)$

d) $\sin\left(\sin^{-1}\frac{3}{5} + \cos^{-1}\left(-\frac{12}{37}\right)\right)$

7) (6 points each) Solve the equation for the variable:

a) $\sqrt{2} \cos x + 1 = 0$:

b) $\sin(2x) = \frac{1}{2}$ on $[0, 2\pi)$

8) (6 points) Fill in the blank using interval notation:

	$\sin x$ *	$\cos x$ *	$\tan x$ *	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain						
Range						

*Write the domain restrictions for these three functions.

9) Short answer:

a) (4 points) Explain why $\sin\left(\sin^{-1} \frac{1}{2}\right) = \frac{1}{2}$ but $\sin^{-1}\left(\sin \frac{7\pi}{4}\right) \neq \frac{7\pi}{4}$.

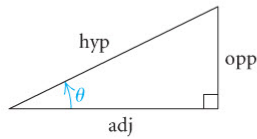
b) Extra credit: (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

c) Extra credit: (2 points) Referring to number 4, find the exact value for $\sin(4\beta)$:

Trigonometry

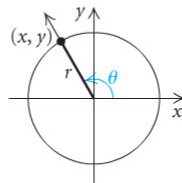
Trigonometric Functions

Acute Angles



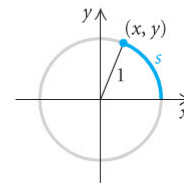
$$\begin{aligned}\sin \theta &= \frac{\text{opp}}{\text{hyp}}, & \csc \theta &= \frac{\text{hyp}}{\text{opp}}, \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}}, & \sec \theta &= \frac{\text{hyp}}{\text{adj}}, \\ \tan \theta &= \frac{\text{opp}}{\text{adj}}, & \cot \theta &= \frac{\text{adj}}{\text{opp}}\end{aligned}$$

Any Angle



$$\begin{aligned}\sin \theta &= \frac{y}{r}, & \csc \theta &= \frac{r}{y}, \\ \cos \theta &= \frac{x}{r}, & \sec \theta &= \frac{r}{x}, \\ \tan \theta &= \frac{y}{x}, & \cot \theta &= \frac{x}{y}\end{aligned}$$

Real Numbers



$$\begin{aligned}\sin s &= y, & \csc s &= \frac{1}{y}, \\ \cos s &= x, & \sec s &= \frac{1}{x}, \\ \tan s &= \frac{y}{x}, & \cot s &= \frac{x}{y}\end{aligned}$$

Basic Trigonometric Identities

$$\begin{aligned}\sin(-x) &= -\sin x, \\ \cos(-x) &= \cos x, \\ \tan(-x) &= -\tan x,\end{aligned}$$

$$\begin{aligned}\tan x &= \frac{\sin x}{\cos x}, \\ \cot x &= \frac{\cos x}{\sin x},\end{aligned}$$

$$\begin{aligned}\csc x &= \frac{1}{\sin x}, \\ \sec x &= \frac{1}{\cos x}, \\ \cot x &= \frac{1}{\tan x}\end{aligned}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1, \\ 1 + \cot^2 x &= \csc^2 x, \\ 1 + \tan^2 x &= \sec^2 x\end{aligned}$$

Identities Involving $\pi/2$

$$\begin{aligned}\sin(\pi/2 - x) &= \cos x, \\ \cos(\pi/2 - x) &= \sin x, & \sin(x \pm \pi/2) &= \pm \cos x, \\ \tan(\pi/2 - x) &= \cot x, & \cos(x \pm \pi/2) &= \mp \sin x\end{aligned}$$

Sum and Difference Identities

$$\begin{aligned}\sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v, \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v, \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}\end{aligned}$$

Double-Angle Identities

$$\begin{aligned}\sin 2x &= 2 \sin x \cos x, \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \\ &= 2 \cos^2 x - 1, \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$

Half-Angle Identities

$$\begin{aligned}\sin \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{2}}, & \cos \frac{x}{2} &= \pm \sqrt{\frac{1 + \cos x}{2}}, \\ \tan \frac{x}{2} &= \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}\end{aligned}$$