

15) (8 points) Solve the following system using elimination:

$$\begin{array}{r} \textcircled{1} \quad 2x + 3y + 2z = -10 \\ \quad -x + 4y - 2z = -23 \\ \hline \quad x + 7y = -33 \end{array}$$

$$\begin{array}{r} \textcircled{2} \quad 4x + 6y + 4z = -20 \\ \quad x + y - 4z = -5 \\ \hline 5x + 7y = -25 \end{array}$$

$$\begin{array}{r} \textcircled{3} \quad -x - 7y = 33 \\ \quad 5x + 7y = -25 \\ \hline 4x = 8 \end{array}$$

$$\textcircled{4} \quad x = 2$$

$$\begin{array}{r} \textcircled{5} \quad 2 + 7y = -33 \\ \quad 7y = -35 \\ \quad y = -5 \end{array}$$

$$\begin{array}{r} 2(2) + 3(-5) + 2z = -10 \\ 2z - 1 = -10 \\ 2z = -9 \Rightarrow z = -\frac{9}{2} \end{array}$$

$$\begin{cases} 2x + 3y + 2z = -10 \\ x + y - 4z = -5 \\ x - 4y + 2z = 23 \end{cases}$$

$$\textcircled{6} \quad \left(2, -5, \frac{1}{2} \right)$$

16) (5 points) Solve the following system using elimination. If you are in a special case, say which case you are in and explain how you know:

$$\begin{array}{r} \textcircled{1} \quad x + 2y + 2z = 4 \\ \quad 4x - 2y + 6z = 20 \\ \hline 5x + 8z = 24 \end{array}$$

$$\begin{array}{r} \textcircled{3} \quad 5x + 8z = 24 \\ \quad -5x - 8z = -25 \\ \hline 0 = -1 \end{array}$$

$$\begin{cases} x + 2y + 2z = 4 \\ x - 3y + z = 5 \\ 2x - y + 3z = 10 \end{cases}$$

False! $0 \neq -1$ No solution!

$$\begin{array}{r} \textcircled{2} \quad x - 3y + z = 5 \\ \quad -6x + 3y - 9z = -30 \\ \hline \quad -5x - 8z = -25 \end{array}$$

17) (3 points part a, 5 points part b) Consider the following problem.



Mike has yet to decide on a new hairstyle while playing the games.

Unable to miss a great deal, Mike goes wild during a video game sale at several stores. At Target, Mike buys 3 copies of *Final Fantasy XV*, 4 copies of *Uncharted 4*, and 2 copies of *No Man's Sky* and spends \$120.38. At Best Buy, Mike buys 2 copies of *Final Fantasy XV*, 3 copies of *Uncharted 4* and one copy of *No Man's Sky* for \$98.75. The price for one copy of *Final Fantasy XV* is \$10 more than twice the cost of *No Man's Sky*. Assume the prices for each game does not change depending on where the game was purchased. How much does each game cost?

a) Name and define variables:

$$\begin{array}{l} x = \text{cost for one copy of FFXV} \\ y = \text{cost for one copy of U4} \\ z = \text{cost for one copy of NMS} \end{array}$$

b) Set up the system but do not solve it.

$$\begin{cases} 3x + 4y + 2z = 120.38 \\ 2x + 3y + 1z = 98.75 \\ x = 10 + 2z \\ \text{or } x - 2z = 10 \end{cases}$$

29

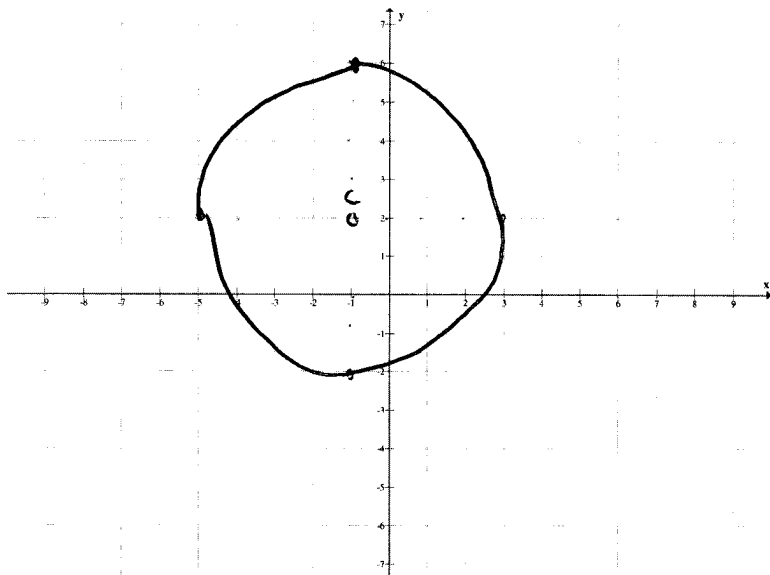
4) (4 points each) Consider the circle $(x+1)^2 + (y-2)^2 = 16$:

a) What is the....

Center? $(-1, 2)$

Radius? 4

b) Sketch a graph:



5) (6 points) Find the equation of the circle in standard form where $(18, 6)$ and $(6, 1)$ are endpoints of a diameter of the circle:

Center: $(\frac{18+6}{2}, \frac{6+1}{2}) = (12, \frac{7}{2})$

$$(x-12)^2 + (y-\frac{7}{2})^2 = (\frac{13}{2})^2 = \frac{169}{4}$$

radius: $\frac{\sqrt{(6-18)^2 + (1-6)^2}}{2} = \frac{\sqrt{144+25}}{2} = \frac{13}{2}$

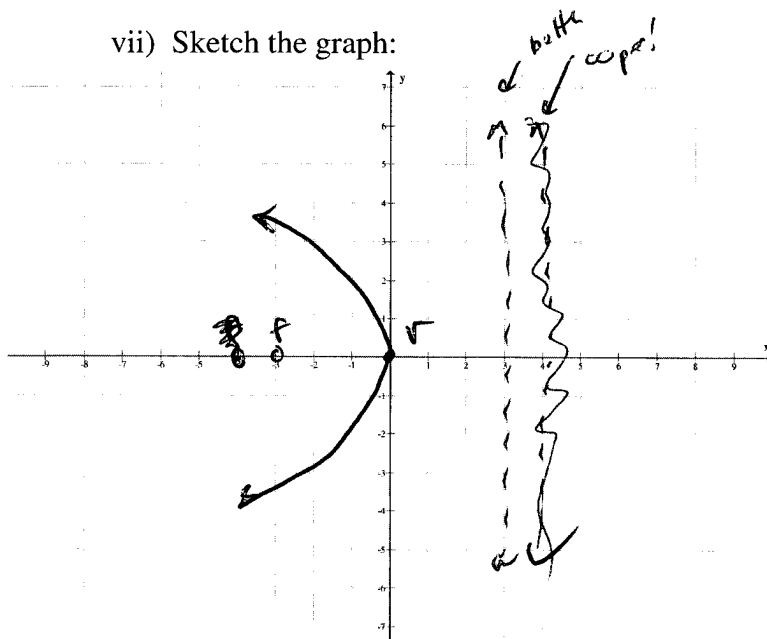
6) (8 points) Fill in the information for the parabola $y^2 = -12x$:

i) $h =$ 0

ii) $k =$ 0

iii) $p =$ -3

vii) Sketch the graph:



Write iv and v as ordered pairs:

iv) Center $(0, 0)$

v) Focus $(-3, 0)$

vi) Directrix $x = 3$

I had coffee today.

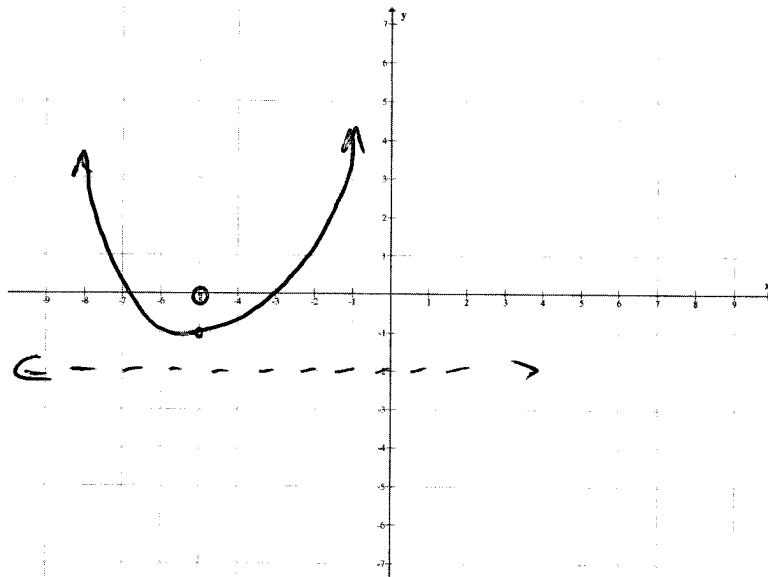
7) (8 points) Fill in the information for the parabola $(x+5)^2 = 4(y+1)$:

- i) $h = \underline{-5}$
 ii) $k = \underline{-1}$
 iii) $p = \underline{1}$

vi) Sketch the graph:

Write iv and v as ordered pairs:

- iv) Center $\underline{(-5, -1)}$
 v) Focus $\underline{(-5, 0)}$
 vi) Directrix $\underline{y = -2}$



8) (5 points each) For the ellipse $9x^2 - 36x + 25y^2 + 150y + 36 = 0$

a) Rewrite the equation by completing the square for both variables:

$$9(x^2 - 4x + 4) + 25(y^2 + 6y + 9) = -36 + 36 + 225$$

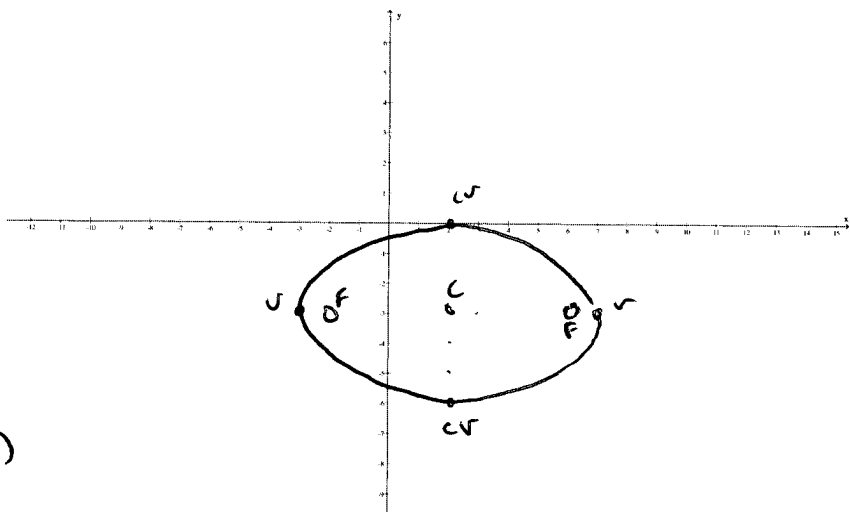
$$9(x-2)^2 + 25(y+3)^2 = 225 \quad \text{divide by } 225$$

$$\boxed{\frac{(x-2)^2}{25} + \frac{(y+3)^2}{9} = 1}$$

b) Find the **exact values** for the following:

- i) $h = \underline{2}$
 ii) $k = \underline{-3}$
 iii) $a = \underline{5}$
 iv) $b = \underline{3}$
 v) $c = \underline{4}$

c) Sketch the graph:



Write as ordered pairs:

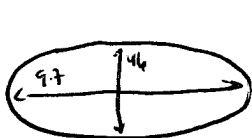
- vi) Center $\underline{(2, -3)}$
 vii) Vertices $\underline{(-3, -3)} \quad \underline{(7, -3)}$
 viii) Co-Vertices $\underline{(2, 0)} \quad \underline{(2, -6)}$
 ix) Foci $\underline{(-2, -3)} \quad \underline{(6, -3)}$

23

9) (7 points) Statuary Hall, also known as the Whispering Gallery, is an elliptical room in the United States Capitol in Washington D.C. where a person standing at one focus of the room can hear even a whisper spoken by a person standing at the other focus. Statuary Hall is 46 feet wide and 97 feet long. Assuming a horizontal ellipse, find...

a) The equation of the ellipse for the room:

b) The location of the foci from the center of the room. Round to two decimal places:



$$\frac{x^2}{(97/2)^2} + \frac{y^2}{23^2} = 1$$

$$\Rightarrow a = \frac{97}{2}$$

$$b = \frac{46}{2} = 23$$

$$\frac{x^2}{9409/4} + \frac{y^2}{529} = 1$$

↳ or
2352.25

$$c^2 = a^2 - b^2$$

$$c = \sqrt{\frac{9409}{4} - 529}$$

$$\approx \boxed{42.70 \text{ ft}}$$

10) (8 points) Fill in the information for the hyperbola $\frac{x^2}{64} - \frac{y^2}{36} = 1$:

i) $h = \underline{0}$

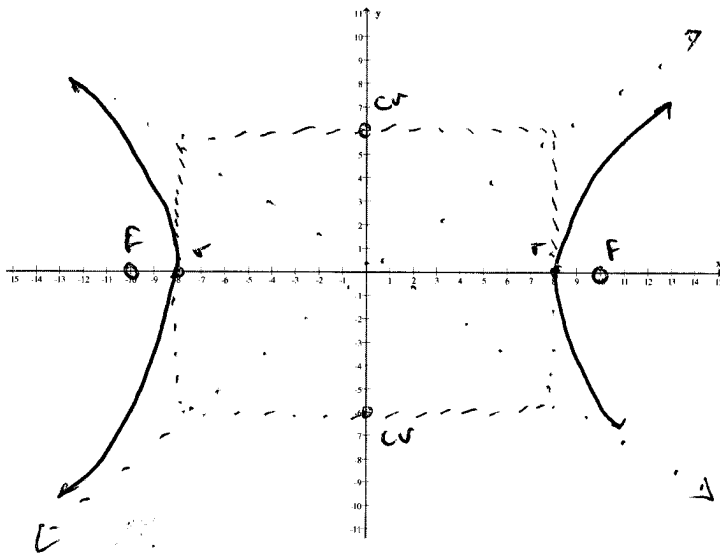
ii) $k = \underline{0}$

iii) $a = \underline{8}$

iv) $b = \underline{6}$

v) $c = \underline{10}$

vi) Sketch the graph:



Write as ordered pairs:

vi) Center $\underline{(0, 0)}$

vii) Vertices $\underline{(-8, 0) (8, 0)}$

viii) Co-Vertices $\underline{(0, 6) (0, -6)}$

ix) Foci $\underline{(10, 0) (-10, 0)}$

11) (8 points each) Solve the following systems:

a) $\begin{cases} x^2 + y^2 = 16 \\ x + 2y = 4 \Rightarrow x = 4 - 2y \end{cases}$

$$(4 - 2y)^2 + y^2 = 16$$

$$16 - 16y + 4y^2 + y^2 = 16$$

$$5y^2 - 16y = 0 \Rightarrow y(5y - 16) = 0$$

$$\Rightarrow y = 0$$

$$y = \frac{16}{5}$$

$$x = 4 - 2(0) = 4$$

$$x = 4 - 2(\frac{16}{5}) = -\frac{12}{5}$$

$$\boxed{(4, 0) \quad (-\frac{12}{5}, \frac{16}{5})}$$

b) $\begin{cases} x^2 - 4y^2 = 4 \\ x^2 + y^2 = 4 \end{cases} \times 4$

$$x^2 - 4y = 4$$

$$4x^2 + 4y^2 = 16$$

$$5x^2 = 20 \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$$\begin{aligned} x &= 2 \\ \cancel{x^2 - 4y^2} &= 4 \\ x^2 + y^2 &= 4 \\ y^2 &= 0 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} x &= -2 \\ (-2)^2 + y^2 &= 4 \\ y^2 &= 0 \Rightarrow y = 0 \end{aligned}$$

$$\boxed{(2, 0) \quad (-2, 0)}$$