

1) (3 points each) Let  $f(x) = \sqrt{x+5}$  and  $g(x) = x^2 + 1$ . Find...

a)  $(f+g)(x)$

$$= \boxed{\sqrt{x+5} + x^2 + 1}$$

b)  $(f-g)(4)$

$$= f(4) - g(4) \\ = 3 - 17 = \boxed{-14}$$

c)  $(f \circ g)(x)$

$$= \sqrt{x^2 + 1 + 5} \\ = \boxed{\sqrt{x^2 + 6}}$$

d)  $(f \circ g)(3)$

$$= \sqrt{3^2 + 6} \\ = \boxed{\sqrt{15}}$$

2) (3 points) Find two functions,  $f$  and  $g$ , such that  $f \circ g = h$  where  $h(x) = \sqrt[5]{2x^2 + 7}$ :

$$f(x) = \sqrt[5]{x} \\ g(x) = 2x^2 + 7$$

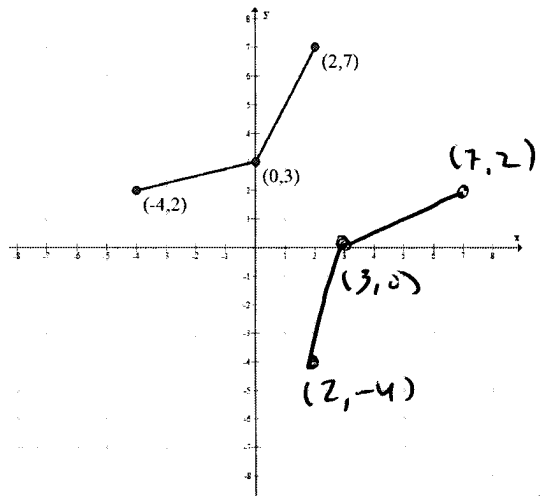
multiple answer

3) (3 points each) For the following one-to-one functions, find the inverse. For part b, draw the inverse on the graph:

a)  $\{(8,6), (3,-2), (0,7), (5,2)\}$

$$\{(6,8), (-2,3), (7,0), (2,5)\}$$

b)



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4) (5 points each) For the function  $f(x) = 2x^5 - 1$ , find (showing all necessary work)...

a)  $f^{-1}(x)$

$$\frac{x+1}{2} = y^5$$

b)  $(f \circ f^{-1})(x) = 2 \left( \sqrt[5]{\frac{x+1}{2}} \right)^5 - 1$

①  $y = 2x^5 - 1$

②  $x = 2y^5 - 1$

③  $x + 1 = 2y^5$

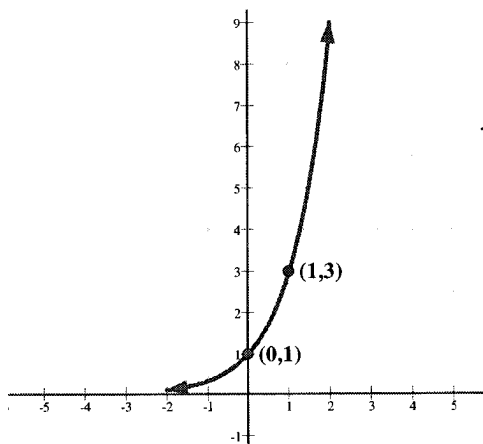
$$\sqrt[5]{\frac{x+1}{2}} = y$$

$$f^{-1}(x) = \sqrt[5]{\frac{x+1}{2}}$$

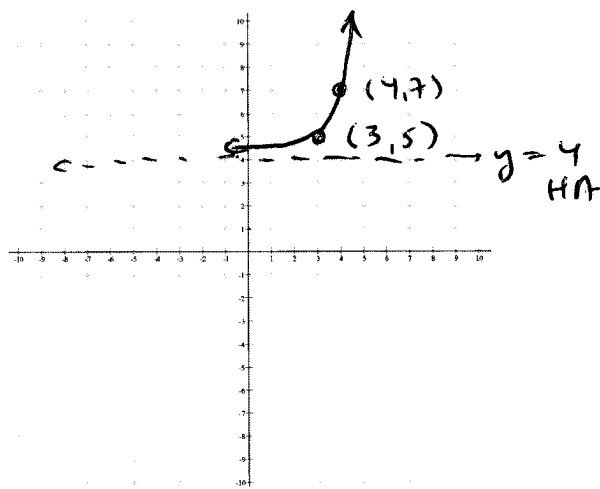
$$= 2 \left( \frac{x+1}{2} \right) - 1$$

$$= x + 1 - 1 = \boxed{x}$$

5) (5 points) Graph  $g(x) = 3^{x-3} + 4$  by transforming the given function  $y = 3^x$ . Be sure to move and label the given points and asymptote.



1) Right 3  
2) up 4



6) (3 points each) Write as a logarithmic equation:

a)  $81^{\frac{1}{4}} = 3$

b)  $e^x = 2$

$$\log_{81} 3 = \frac{1}{4}$$

$$\ln 2 = x$$

7) (3 points each) Write as an exponential equation.

a)  $\log_2 32 = 5$

b)  $\log 1000 = 3$

$$2^5 = 32$$

$$10^3 = 1000$$

~~scribble~~  
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8) (2 points each) Simplify without a calculator:

a)  $\log_9 81$

$$2$$

b)  $\log_{16} 2$

$$\frac{1}{4}$$

c)  $\log_5 \sqrt[4]{5}$

$$\frac{1}{4}$$

d)  $\ln e^{-4}$

$$-4$$

9) (3 points) Fill in the blank: Jo Jo Ba decides to put away her javelins and take up teaching math. One day, she writes on the board  $\ln 46 - \log_3 4$  and says "Class, this is correctly pronounced questa non è la risposta"

10) (5 points) Write  $8\log_3 x + 3\log_3 y - 10\log_3 z$  as one logarithm:

$$\begin{aligned} &= \log_3 x^8 + \log_3 y^3 - \log_3 z^{10} \\ &= \log_3 (x^8 y^3) - \log_3 z^{10} = \log_3 \left( \frac{x^8 y^3}{z^{10}} \right) \end{aligned}$$

11) (6 points) Given that  $\ln x = -3$ ,  $\ln y = 8$ , and  $\ln z = 2$ , find the exact value for  $\ln \left( \frac{x^3 y}{z^3} \right)$ :

$$\begin{aligned} \ln \frac{x^3 y}{z^3} &= \ln x^3 y - \ln z^3 = \ln x^3 + \ln y - \ln z^3 \\ &= 3 \ln x + \ln y - 3 \ln z \\ &= 3(-3) + 8 - 3(2) = \boxed{-7} \end{aligned}$$

12) (6 points each) Solve for the variable. Find the exact value (no decimals):

a)  $4^{3x+2} = 64$

$$4^{3x+2} = 4^3$$

$$3x+2 = 3$$

$$\boxed{x = -\frac{1}{3}}$$

b)  $12^{x-3} = 4$

$$\log_{12} 4 = x-3$$

$$\boxed{x = \log_{12} 4 + 3}$$

$$\underline{=} \frac{\log 4}{\log 12} + 3$$

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13) (6 points each) Solve for the variable. Find the exact value (no decimals):

a)  $\log_5(5x-1) = \log_5(2x+6)$

$$5x-1 = 2x+6$$

$$3x = 7$$

$$x = \frac{7}{3}$$

b)  $\log_2(2x+1) - \log_2(x-5) = 3$

$$\log_2 \frac{2x+1}{x-5} = 3$$

$$\frac{2x+1}{x-5} = 2^3 = 8$$

$$2x+1 = 8x-40$$

$$41 = 6x \rightarrow$$

$$x = \frac{41}{6}$$

14) (6 points each) Suppose that \$5,000 was deposited into an account that offers a 3.75% annual interest compounded continuously. How long will it take for the account to reach \$8,000? Use the formula  $A = Pe^{rt}$ .

$$8000 = 5000 e^{.0375t}$$

$$\frac{8}{5} = e^{.0375t}$$

$$\ln \frac{8}{5} = .0375t$$

$$t = \frac{\ln \frac{8}{5}}{.0375}$$

$$t \approx 12.53 \text{ year}$$

Extra Credit: (1 point each) Fill in the blank:

a)  $\log_b x = y$  implies \_\_\_\_\_

b)  $\log_b(MN) =$  \_\_\_\_\_

c)  $\log_b \frac{M}{N} =$  \_\_\_\_\_



d)  $p \cdot \log_b M =$  \_\_\_\_\_

e)  $b^{\log_b M} =$  \_\_\_\_\_

f)  $\log_b b^M =$  \_\_\_\_\_

g)  $\log_b 1 =$  \_\_\_\_\_

h)  $\log_b b =$  \_\_\_\_\_

i)  $b^M = b^N$  implies \_\_\_\_\_

j)  $\log_b M = \log_b N$  implies \_\_\_\_\_

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