1) (3 points each) Solve for the variable and write your answer in interval notation:

a) $5x+7 \geq 2x-12$

$3x \geq -19$

$x \geq -\frac{19}{3}$

$\left[-\frac{19}{3}, \infty\right)$

b) $4 < 2x+3 \leq 11$

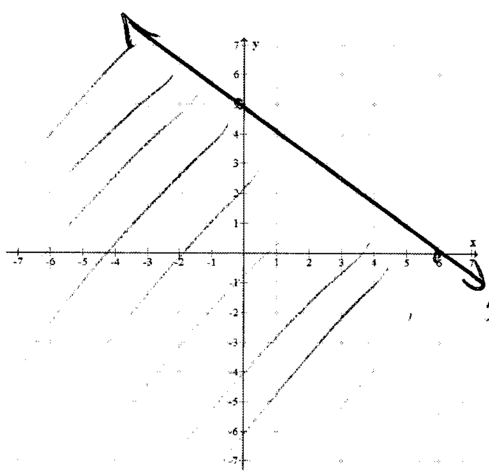
$1 < 2x \leq 8$

$\frac{1}{2} < x \leq 4$

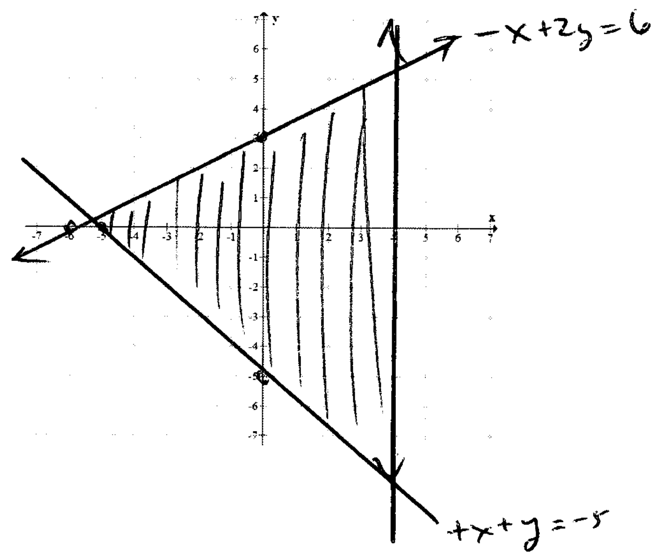
$\left(\frac{1}{2}, 4\right]$

2) Sketch a graph of the following inequalities. Be sure to shade your final answer the darkest:

a) (3 points) $5x+6y \leq 30$



b) (5 points) $\begin{cases} -x+2y \leq 6 \\ x \leq 4 \\ x+y \geq -5 \end{cases}$



3) (6 points) For the following situation, define the necessary variables and write the corresponding system of equations. Give three ordered pairs that satisfy the system.

Michele di San Gimignano, a gelateria in northern Italy, offers several gourmet gelatos. Two of the top selling gelatos are the chocolate hazelnut and the coffee flavors. Each two-quart batch of the chocolate hazelnut gelato needs 2 cups of heavy cream, 1 cup of sugar, and 6 eggs. Each two-quart batch of the coffee gelato requires 1 cup of heavy cream, 2 cups of sugar, and 6 eggs. On a particular day, the gelateria has at most 13 cups of heavy cream, 14 cups of sugar, and 48 eggs available.

x = # of batches of chocolate hazelnut

y = # of batches of coffee

Heavy cream
sugar
eggs

$\begin{cases} 2x + 1y \leq 13 \\ 1x + 2y \leq 14 \\ 6x + 6y \leq 48 \\ x \geq 0 \\ y \geq 0 \end{cases}$

many sol'n

(0,0) → (0,7)
(1,0) → (1,6)
(2,0) → (2,6)
(3,0) → (3,5)
(4,0) → (4,4)
(5,0) → (5,3)
(6,0) → (6,1)

30

4) (4 points each) Divide:

a) $\dfrac{4a^2b^3 + 12ab^4 - 7a^3b^2}{2a^2b^2}$

$\dfrac{4a^2b^3}{2a^2b^2} + \dfrac{12ab^4}{2a^2b^2} - \dfrac{7a^3b^2}{2a^2b^2}$

$= \boxed{2b + \dfrac{6b^2}{a} - \dfrac{7a}{2}}$

b) $\dfrac{4x^3 + 2x^2 - x + 1}{x^2 + x + 2}$

$$\begin{array}{r} 4x - 2 \\ x^2+x+2 \overline{)\, 4x^3 + 2x^2 - x + 1} \\ -(4x^3 + 4x^2 + 8x) \\ \hline -2x^2 - 9x + 1 \\ -(-2x^2 - 2x - 4) \\ \hline -7x + 5 \end{array}$$

c) $\dfrac{3x^4 + 5x^3 - 2x + 6}{x-2}$ (Use Synthetic Division)

$$\begin{array}{r|rrrrr} 2 & 3 & 5 & 0 & -2 & 6 \\ & \downarrow & 6 & 22 & 44 & 84 \\ \hline & 3 & 11 & 22 & 42 & 90 \end{array}$$

$\boxed{3x^3 + 11x^2 + 22x + 42 \quad R\ 90}$

5) (2 points) Referring to 4c, was $x-2$ a factor of $3x^4 + 5x^3 - 2x + 6$? Why or why not?

Nope! ¡c'é un resto.

6) (3 points each) Determine if the following are functions. If so, identify their domain and range. If not, explain why not.

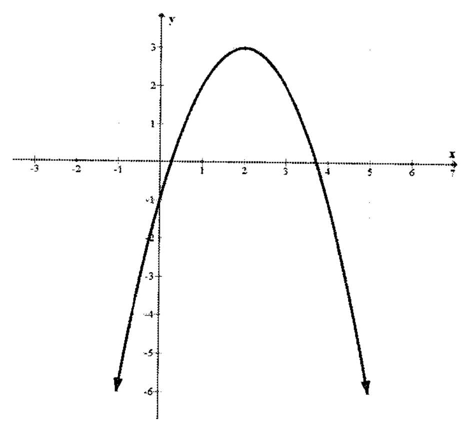
a) $\{(4,5), (-2,3), (0,12), (1,9)\}$

Function!

D: $\{4, -2, 0, 1\}$

R: $\{5, 3, 12, 9\}$

b)



Function

D: $\mathbb{R}$

R: $(-\infty, 3]$

c) *Thirty days hath September, April, June, and November. Thirty-one hath all the rest, Except for February clear which has 28 and 29 in a leap year.*

Not a function! Feb $\rightarrow 28$, $\rightarrow 29$ SAD PANDA

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7) (3 points each) For the functions $f(x) = 2x + 5$ and $g(x) = 3x^2 - 5$, find and simplify…

a) $f(4)$ $= 2(4) + 5$

$= 13$

b) $g(2)$ $= 3(2)^2 - 5$

$= 7$

c) $f(-1) + g(3)$

$2(-1) + 5 + 3(3)^2 - 5$

$= 25$

d) $g(x+h)$ $= 3(x+h)^2 - 5$

$= 3(x^2 + 2xh + h^2) - 5$

$= 3x^2 + 6xh + 3h^2 - 5$

8) (4 points each) Find the domain of the following functions:

a) $f(x) = 6x^3 + 18x^2 - x - 3$

all the reals

b) $g(x) = \dfrac{2x+1}{4x^2 + 5x - 6}$

$4x^2 + 5x - 6 = 0$

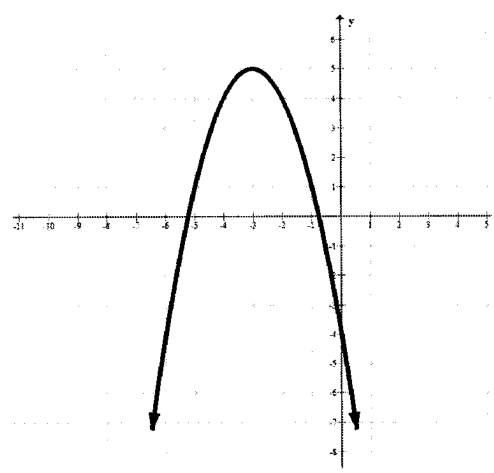
$(4x - 3)(x + 2) = 0$

$x \neq -2, 3/4$

9) (2 points) Short Answer: Explain why you excluded certain numbers in the problem 8b:

cuz math

10) (3 points) Determine the equation of the function shown in the graph to the right:



$f(x) = -(x+3)^2 + 5$

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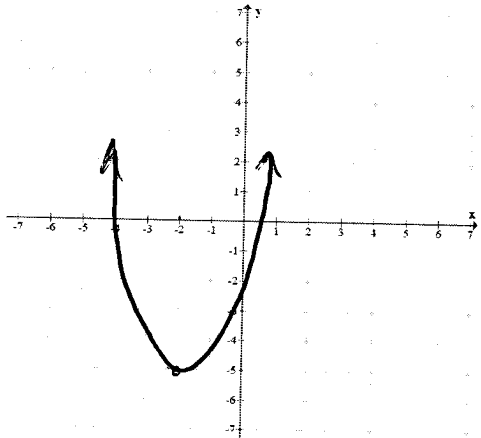
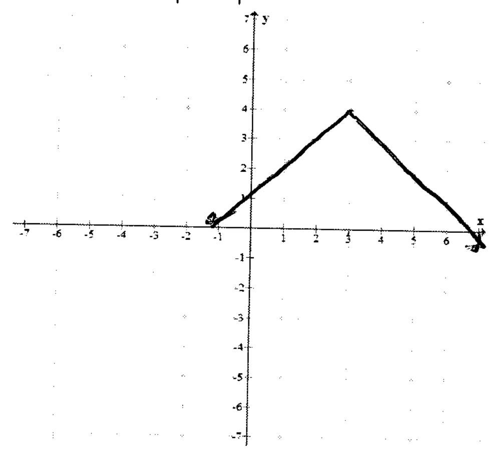
11) (5 points each) For each of the functions, explain the steps necessary to sketch the graph and then sketch the graph:

a) $f(x)=(x+2)^2-5$



① Left 2
② Down 5

b) $g(x)=-|x-3|+4$



① Right 3
② Reflect vertically
③ Up 4

12) (2 points each) For the sets $A=\{1,2,3,4,5,6\}$, $B=\{3,5,7,9\}$, and $C=\{x \mid x \text{ is an even natural number}\}$, find...

a) $A \cup B$

$\{1,2,3,4,5,6,7,9\}$

b) $B \cap C$

$\varnothing$

c) $A \cap B$

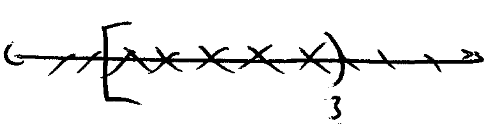
$\{3,5\}$

13) (4 points each) Solve the following compound inequalities. Be sure to draw the necessary number lines. Write answer in interval notation:

a) $5x+3 \le 7x+9$ and $8x+1<13$

$-2x \le 6$ $\quad$ $8x<12$

$x \ge -3$ and $x<\frac{3}{2}$



$[-3, \frac{3}{2})$

b) $7-2x \le 3$ or $6x+3<-12+x$

$-2x \le -4$ $\quad$ $5x<-15$

$x \ge 2$ or $x<-3$

$(-\infty,-3) \cup [2,\infty)$

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14) (3 points each) Solve for the variable. For part b, write your answer in interval notation:

a) $4-\dfrac{|x+3|}{5}=-2$

$$-\frac{|x+3|}{5}=-6$$

$$|x+3|=30$$

$x+3=30$ $\quad$ $-x-3=30$

$x=27$ $\quad$ $x=-33$

b) $-3|2x+1|+5>-10$

$$-3|2x+1|>-15$$

$$|2x+1|<5 \quad \text{(and)}$$

$2x+1<5$ and $-2x-1<5$

$x<2$ and $x>-3$

$(-3,2)$

15) (2 points) Explain why $|x+4|\geq -5$ is always true and why $|x-6|\leq -3$ is always false:

fancy mathy words go hither