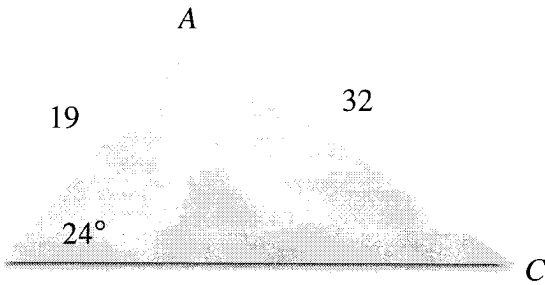


1) (6 points each) Solve the following triangles using the appropriate Law. Be sure to show all necessary work. Round answers to nearest whole number:

a)



$$A = \underline{142^\circ} \quad a = \underline{48}$$

$$B = \underline{24^\circ} \quad b = \underline{32}$$

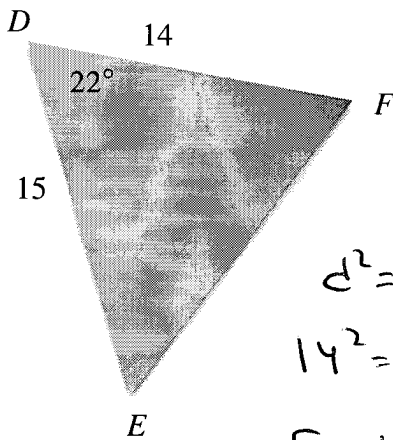
$$C = \underline{14^\circ} \quad c = \underline{19}$$

$$\frac{\sin C}{19} = \frac{\sin 24^\circ}{32} \Rightarrow C = \sin^{-1}\left(\frac{19 \sin 24^\circ}{32}\right) \approx 14^\circ$$

$$A = 180 - (24 + 14) = 142^\circ$$

$$\frac{a}{\sin 142^\circ} = \frac{32}{\sin 24^\circ} \Rightarrow a = \frac{32 \sin 142^\circ}{\sin 24^\circ} \approx 48$$

b)



$$D = \underline{22^\circ} \quad d = \underline{6}$$

$$E = \underline{69^\circ} \quad e = \underline{14}$$

$$F = \underline{89^\circ} \quad f = \underline{15}$$

$$d^2 = 14^2 + 15^2 - 2(14)(15)\cos 22^\circ \Rightarrow d = 6$$

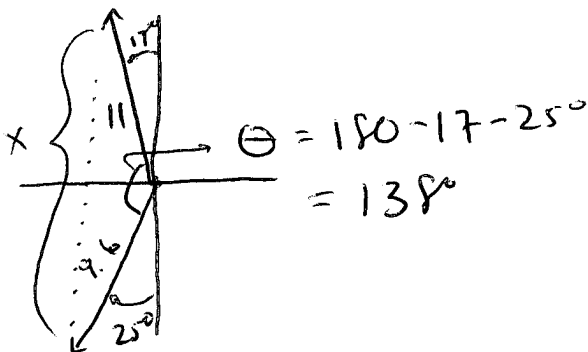
$$14^2 = 6^2 + 15^2 - 2(6)(15)\cos E \Rightarrow E = \cos^{-1}\left(\frac{14^2 - 6^2 - 15^2}{-2 \cdot 6 \cdot 15}\right) \approx 69^\circ$$

$$F = 180 - 22 - 69 = 89^\circ$$

2) (2 points each) Concerning the given information of a triangle, how do you know when to use the Law of Sines versus the Law of Cosines?



3) (5 points) Mike announces a test and two students begin to run away from Mike from the same point. One student runs with a bearing of $S25^\circ W$ at 4.8 mph while the other student runs with a bearing of $N17^\circ W$ at 5.5 mph. How far are the students from each other after 2 hours? Round answer to two decimal places. Draw a picture for this scenario.



$$x = \sqrt{11^2 + 9.6^2 - 2(11)(9.6)\cos 138^\circ}$$

$$x \approx \underline{19.24 \text{ mi}}$$

19

4) (4 points each) Convert...

a) $7\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$ to standard form:

$$= 7\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= \boxed{\frac{7}{2} - \frac{7\sqrt{3}}{2}i}$$

b) $-6\sqrt{3} - 6i$ to trigonometric form:

$$r = \sqrt{(-6\sqrt{3})^2 + (-6)^2} = \sqrt{144} = 12$$

$$\left. \begin{aligned} \cos\theta &= \frac{-6\sqrt{3}}{12} = -\frac{\sqrt{3}}{2} \\ \sin\theta &= \frac{-6}{12} = -\frac{1}{2} \end{aligned} \right\} \theta = 210^\circ \text{ or } \frac{7\pi}{6}$$

$$= 12(\cos 210^\circ + i\sin 210^\circ) \text{ or radian.}$$

5) (4 points each) For the complex numbers $z_1 = \frac{7}{2} - \frac{7\sqrt{3}}{2}i$ and $z_2 = -6\sqrt{3} - 6i$, find the following,

using the trigonometric forms and the formula $z_1 \times z_2 = r_1 \times r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$ for

part a and $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$ for part b.

a) $z_1 \times z_2$

$$= 7 \cdot 12 (\cos(\frac{5\pi}{3} + \frac{7\pi}{6}) + i\sin(\frac{5\pi}{3} + \frac{7\pi}{6}))$$

$$= 84 (\cos \frac{17\pi}{6} + i\sin \frac{17\pi}{6})$$

$$= 84 (\cos \frac{5\pi}{6} + i\sin \frac{5\pi}{6})$$

$$= 84 (-\frac{\sqrt{3}}{2} + \frac{1}{2}i) = \boxed{-42\sqrt{3} + 42i}$$

b) $\frac{z_1}{z_2} = \frac{7}{12} (\cos(\frac{5\pi}{3} - \frac{7\pi}{6}) + i\sin(\frac{5\pi}{3} - \frac{7\pi}{6}))$

$$= \frac{7}{12} (\cos \frac{\pi}{2} + i\sin \frac{\pi}{2})$$

$$= \frac{7}{12} (0 + 1i)$$

$$= \boxed{\frac{7}{12}i}$$

6) (4 points part a; 6 points part b) For the complex number $-4 + 4\sqrt{3}i = 8(\cos 120^\circ + i\sin 120^\circ)$,

find the following. For part a, use the formula $(a + bi)^n = r^n [\cos(n\theta) + i\sin(n\theta)]$. For part b,

use the formula $(a + bi)^{\frac{1}{n}} = r^{\frac{1}{n}} [\cos(\frac{\theta}{n} + \frac{360^\circ}{n} \cdot k) + i\sin(\frac{\theta}{n} + \frac{360^\circ}{n} \cdot k)]$. Write answers in standard form.

a) $(-4 + 4\sqrt{3}i)^2$

$$= 8^2 (\cos(2 \cdot 120^\circ) + i\sin(2 \cdot 120^\circ))$$

$$= 64 (\cos 240^\circ + i\sin 240^\circ)$$

$$= 64 (-\frac{1}{2} - \frac{\sqrt{3}}{2}i)$$

$$= \boxed{-32 - 32\sqrt{3}i}$$

b) The cube roots of $-4 + 4\sqrt{3}i$

$$8^{\frac{1}{3}} [\cos(\frac{120^\circ}{3} + k \cdot \frac{360^\circ}{3}) + i\sin(\frac{120^\circ}{3} + k \cdot \frac{360^\circ}{3})]$$

$$= 2 (\cos(40 + 120^\circ k) + i\sin(40 + 120^\circ k))$$

$$\left. \begin{aligned} k=0 & 2(\cos 40^\circ + i\sin 40^\circ) \\ k=1 & 2(\cos 160^\circ + i\sin 160^\circ) \\ k=2 & 2(\cos 280^\circ + i\sin 280^\circ) \end{aligned} \right\}$$

leave in this form.

\sum_3^6

7) (4 points) Convert the rectangular point $(-5, -5)$ to the polar format (r, θ) :

$$r = \sqrt{(-5)^2 + (-5)^2} = 5\sqrt{2}$$

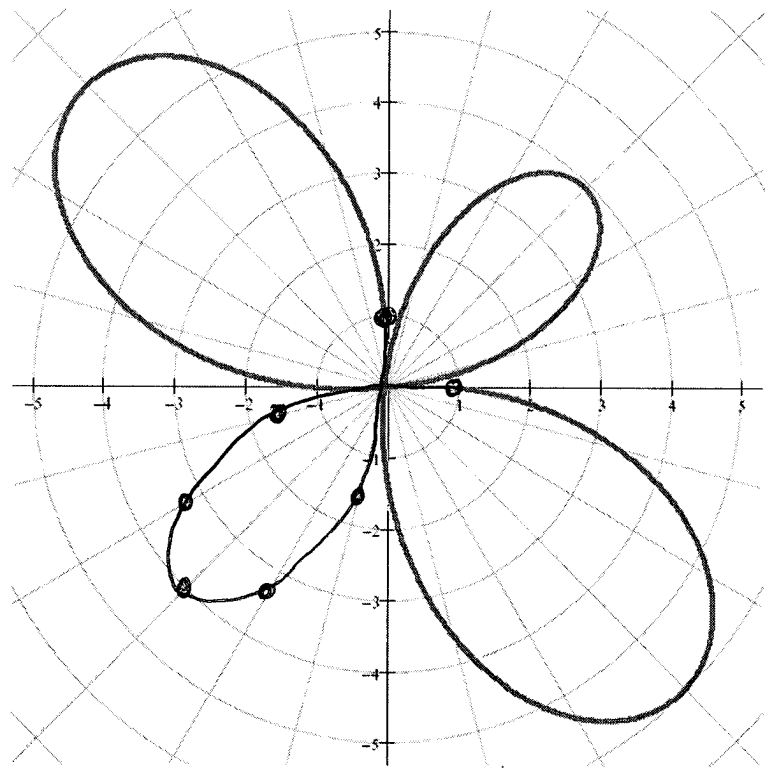
$$\left. \begin{aligned} \cos \theta &= \frac{x}{r} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2} \\ \sin \theta &= \frac{y}{r} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{aligned} \right\}$$

$$\theta = 225^\circ \text{ or } \frac{5\pi}{4}$$

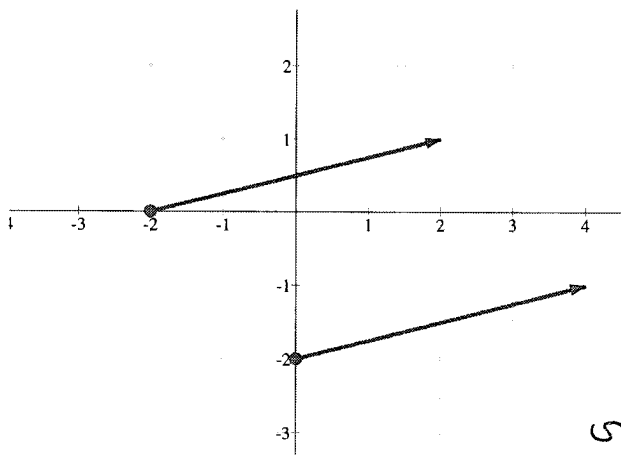
$$(5\sqrt{2}, 225^\circ) \text{ or } \frac{5\pi}{4}$$

8) (6 points) Finish the polar graph of $r = 1 - 5 \sin(2\theta)$ by filling in the chart and plotting points. Round answers to one decimal place:

θ	r
0°	1
15°	-1.5
30°	-3.3
45°	-4
60°	-3.3
75°	-1.5
90°	1



9) (4 points) For the given vectors, determine algebraically if they are equivalent by finding their magnitude and direction:



$$|\vec{u}| = \sqrt{(2 - (-2))^2 + (1 - 0)^2} = \sqrt{17}$$

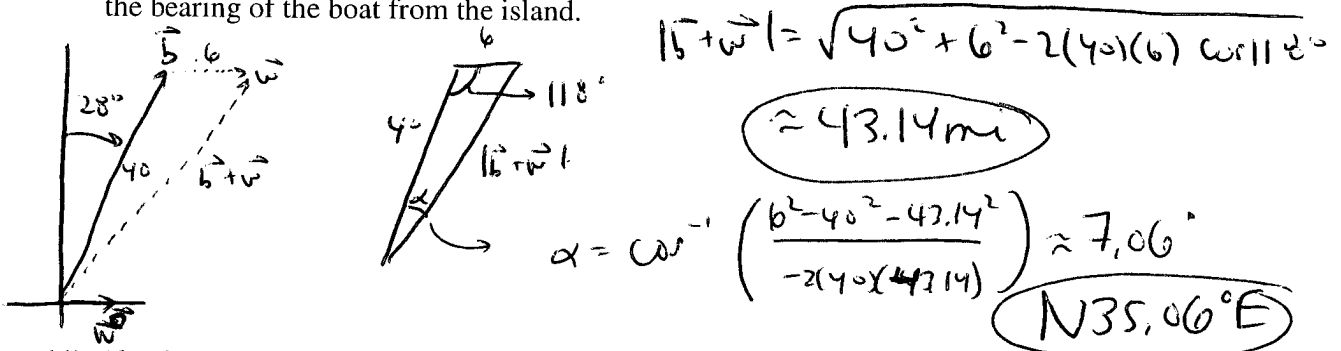
$$|\vec{v}| = \sqrt{(4 - 0)^2 + (-1 - (-2))^2} = \sqrt{17}$$

$$m_{\vec{u}} = \frac{1-0}{2-(-2)} = \frac{1}{4} \quad m_{\vec{v}} = \frac{-1-(-2)}{4-0} = \frac{1}{4}$$

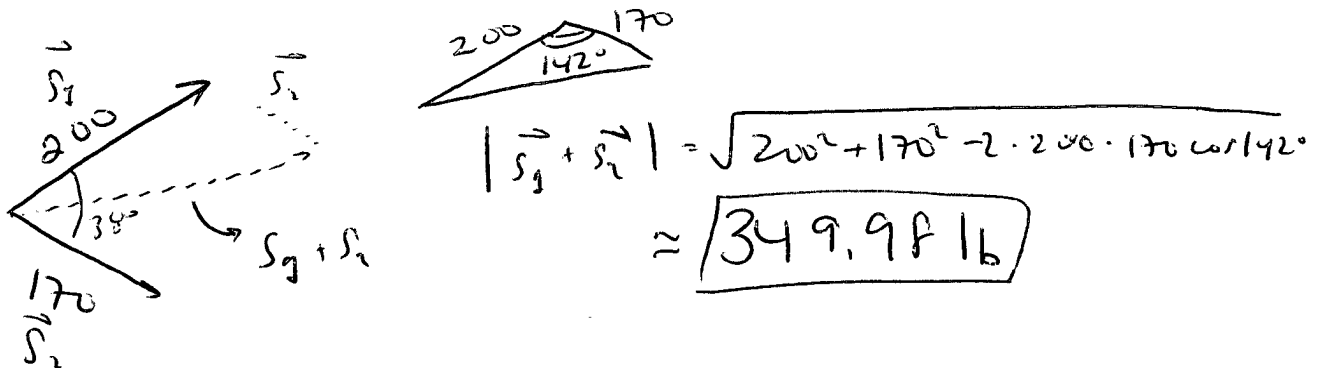
$$\therefore \vec{u} = \vec{v}$$

or we can show they both have component form $\langle 4, 1 \rangle$

- 10) (10 points) Link is traveling in a boat on a bearing of $N28^\circ E$ from Nintendo Island to his new home of Sony Land. His boat travels at a rate of 20 mph. The water has a current from the west at a rate of 3 mph. After 2 hours, how far is Link from Nintendo Island? Also determine the bearing of the boat from the island.



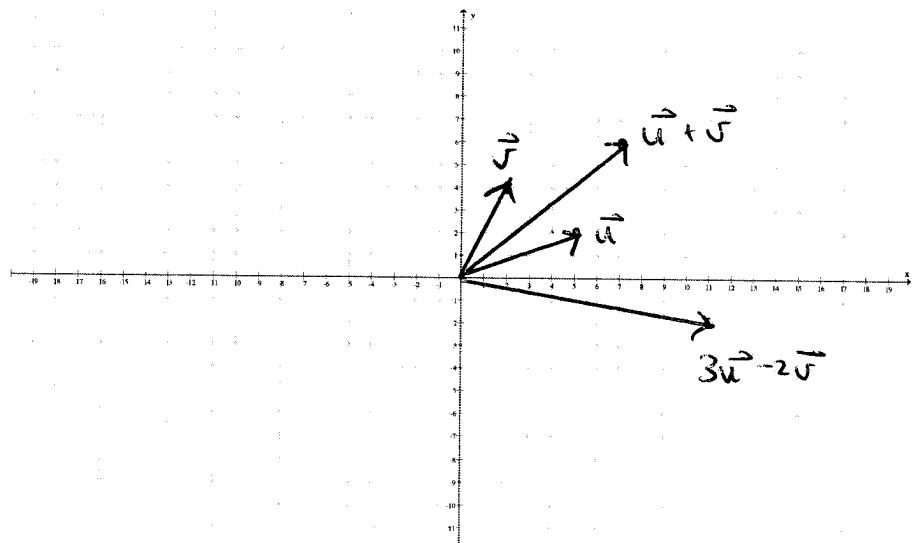
- 11) (6 points) In a fit of nerd rage, two storm troopers draw an unruly George Lucas to his movie trailer when started to complain that there wasn't enough Jar Jar in the new *Star Trek* movie. The one storm trooper pulls with a force of 200 pounds and the other pulls with a force of 170 pounds. The angle between the storm troopers is 38° . Determine the combined force of the troopers in pounds.



- 12) (5 points) Given the vector $\vec{u} = \langle 5, 2 \rangle$ and $\vec{v} = \langle 2, 4 \rangle$, draw the vectors \vec{u} , \vec{v} , $\vec{u} + \vec{v}$, and $3\vec{u} - 2\vec{v}$:

$$\vec{u} + \vec{v} = \langle 7, 6 \rangle$$

$$3\vec{u} - 2\vec{v} = \langle 11, -2 \rangle$$



13) (2 points each) Let $\vec{u} = \langle 4, -3 \rangle$ and $\vec{v} = \langle 1, 2 \rangle$. Find and simplify:

a) $4\vec{u} - 6\vec{v}$

$$= 4\langle 4, -3 \rangle - 6\langle 1, 2 \rangle$$

$$= \langle 16, -12 \rangle + \langle -6, -12 \rangle$$

$$= \langle 10, -24 \rangle$$

b) $|4\vec{u} - 6\vec{v}|$

$$= \sqrt{10^2 + (-24)^2}$$

$$= 26$$

c) The unit vector in the same direction as $4\vec{u} - 6\vec{v}$:

$$= \frac{\langle 10, -24 \rangle}{26} = \left\langle \frac{5}{13}, -\frac{12}{13} \right\rangle$$

d) $\vec{u} \cdot \vec{v}$:

$$= 4 \cdot 1 + (-3) \cdot 2$$

$$= 4 - 6 = -2$$

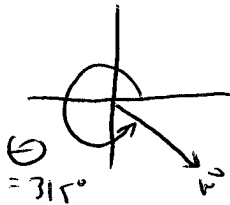
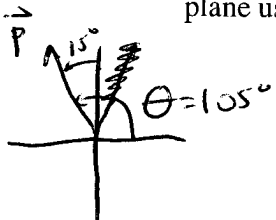
e) The angle between the vectors \vec{u} and \vec{v} . Round to two decimal places. Use the formula

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|}$$

$$\cos \theta = \frac{-2}{\sqrt{25} \cdot \sqrt{5}} \Rightarrow \theta = \cos^{-1}\left(\frac{-2}{12.5}\right)$$

$$\Rightarrow \theta \approx 100.30^\circ$$

14) (10 points) An airplane travels on a bearing of $N15^\circ W$ with an airspeed of 600 mph. A wind is blowing **from the northwest** direction at a speed of 40 mph. Find the ground speed of the plane using the formula $\vec{v} = |\vec{v}|(\cos \theta \vec{i} + \sin \theta \vec{j})$. Round to two decimal places:



$$\vec{p} = 600(\cos 105^\circ \vec{i} + \sin 105^\circ \vec{j})$$

$$\vec{w} = 40(\cos 315^\circ \vec{i} + \sin 315^\circ \vec{j})$$

$$|\vec{p} + \vec{w}| = \sqrt{(600 \cos 105^\circ + 40 \cos 315^\circ)^2 + (600 \sin 105^\circ + 40 \sin 315^\circ)^2}$$

$$\approx 565.71 \text{ mph}$$