1) (4 points each) Simplify:

a) $\tan x(\tan x+\cot x)$

$= \tan^2 x + \overbrace{\tan x \cdot \cot x}^{1}$

$= \tan^2 x + 1$

$= \boxed{\sec^2 x}$

b) $\dfrac{\tan x}{\tan x+\cot x}$

$$= \frac{\frac{\sin x}{\cos x} \cdot \sin x \cos x}{\sin x \cos x \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \sin x \cos x}$$

$$= \frac{\sin^2 x}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x}{1}$$

$= \boxed{\sin^2 x}$

2) (4 points each) Find the exact value of $\tan\dfrac{\pi}{12}$ using the given methods. Rationalize the denominator in part *a*:

a) A Sum or Difference Formula:

$\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$$= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{(\sqrt{3}-1)(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$

$$= \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1-2} = \boxed{2-\sqrt{3}}$$

b) A Half Angle Formula:

$\tan\frac{\pi}{12} = \tan\left(\frac{\pi/6}{2}\right) = \frac{1-\cos\pi/6}{\sin\pi/6}$

$$= \frac{\left(1 - \sqrt{3}/2\right)\cdot 2}{1/2 \cdot 2} = \boxed{2-\sqrt{3}}$$

c) Using your answer from either part *a* or *b* above, explain how you can find the exact value of $\tan\left(\dfrac{13\pi}{12}\right)$ by using $\dfrac{\pi}{12}$ as a reference angle:

$\tan\frac{13\pi}{12} = 2-\sqrt{3}$ because
1) $13\pi/12$ is in QIII
2) Tangent is positive there w/ $\pi/12$ as the reference angle

3) (5 points) Simplify:

$$\frac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta - (\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{\cos\alpha\cos\beta - \sin\alpha\sin\beta - (\cos\alpha\cos\beta + \sin\alpha\sin\beta)} = \frac{2\cos\alpha\cancel{\sin\beta}}{-2\sin\alpha\cancel{\sin\beta}}$$

$= \boxed{-\cot\alpha}$

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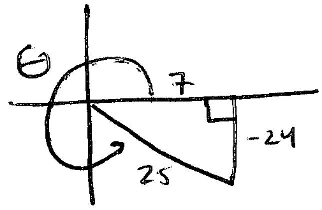

4) (5 points each) Given $\cos\theta = \frac{7}{25}$, where θ is in Quadrant IV, find the exact values for…

a) $\sin(2\theta)$

$= 2\sin\theta\cos\theta$

$= 2\left(-\frac{24}{25}\right)\left(\frac{7}{25}\right) = \frac{-336}{625}$

b) $\cos(2\theta)$

$= 2\cos^2\theta - 1$

$= 2\left(\frac{7}{25}\right)^2 - 1 = \boxed{\frac{-527}{625}}$

c) $\tan(2\theta)$

$= \frac{\sin 2\theta}{\cos 2\theta} = \frac{-336/625}{-527/625}$

$= \boxed{\frac{336}{527}}$

d) The Quadrant where 2θ resides:

Q3

$\sin 2\theta < 0$
$\tan 2\theta > 0$

5) (5 points) Simplify:

$$\frac{\sin^2\alpha}{\tan^2\alpha} - \frac{\tan^2\alpha}{\sec^2\alpha}$$

$= \frac{\sin^2\alpha}{\frac{\sin^2\alpha}{\cos^2\alpha}} - \frac{\frac{\sin^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha}} = \cos^2\alpha - \sin^2\alpha = \boxed{\cos(2\alpha)}$

flip & multiply

6) (4 points each) Simplify or explain why it does not exist:

a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \boxed{\frac{5\pi}{6}}$

b) $\cos\left(\cos^{-1}(-1.1)\right)$ DNE

$-1.1 \notin [-1, 1]$

c) $\cos^{-1}\left(\underbrace{\cos\left(-\frac{2\pi}{3}\right)}\right)$

$\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

d) $\sin\left(\underbrace{\sin^{-1}\left(-\frac{4}{5}\right)}_{u} - \underbrace{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)}_{v}\right)$

$= \sin\left(\sin^{-1}\left(-\frac{4}{5}\right)\right)\cos\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) - \cos\left(\sin^{-1}\left(-\frac{4}{5}\right)\right)\sin\left(\underbrace{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)}_{\frac{5\pi}{6}}\right)$

$= \left(-\frac{4}{5}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{3}{5}\right)\left(\frac{1}{2}\right)$

$= \boxed{\frac{4\sqrt{3} - 3}{10}}$

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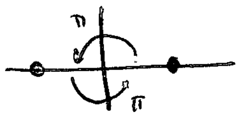
7) (6 points each) Solve for the variable:

a) $\cos^2 x - 1 = 0$

$\cos^2 x = 1 \Rightarrow \cos x = \pm 1$

$\boxed{x = 0 + k \cdot \pi,\ k \in \mathbb{Z}}$



b) $\sin(2x) = \dfrac{\sqrt{3}}{2}$ on $[0, 2\pi)$

$2x = \frac{\pi}{3} + 2\pi k \Rightarrow x = \frac{\pi}{6} + \pi k$

$2x = \frac{2\pi}{3} + 2\pi k \Rightarrow x = \frac{\pi}{3} + \pi k$

$x \in \left\{ \underbrace{\frac{\pi}{6}, \frac{\pi}{3}}_{k=0}, \underbrace{\frac{7\pi}{6}, \frac{4\pi}{3}}_{k=1} \right\}$

8) Fill in the blank using interval notation:

	$\sin x$*	$\cos x$*	$\tan x$*	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain						
Range						

This question intentionally left blank.

*Write the domain restrictions for these three functions.

9) (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

cuz tacos

Extra Credit:

Simplify: $-\sin\left(\frac{\pi}{3} - \alpha\right)\sin\left(\frac{\pi}{3} + \alpha\right) + \cos\left(\frac{\pi}{3} - \alpha\right)\cos\left(\frac{\pi}{3} + \alpha\right)$

$= \cos\left(\frac{\pi}{3} - \alpha + \frac{\pi}{3} + \alpha\right) = \cos\left(\frac{2\pi}{3}\right)$

$= \boxed{-\frac{1}{2}}$

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