MATH 1540 Fall 2016 Mike McCraith
Exam 2 October 27, 2016

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO

- Write your name below on the space provided.
- This test has a total of 6 pages.
- Work the problem in the space provided. If you need more space, write on the back of the test.
- To insure maximum credit, show your work. In general, full credit will not be given for unsupported answers.
- Look only at your test. Don't give me the impression that you are cheating.
- Be sure to write neatly. If I cannot read what was written, do not expect the problem to be graded. A pencil must be used on all tests. Otherwise, the test will not be graded.
- If you finish early, go over the test again.

Good luck!

Number	Maximum	Score
1	8	
2	12	
3	5	
4	20	
5	5	
6	16	
7	12	
8	5	
9	2	
10	15	
Total	100	

Name _____

CIRCLE FINAL ANSWERS

1) (4 points each) Simplify:

a) $\tan x\left(\tan x+\cot x\right)$

b) $\dfrac{\tan x}{\tan x+\cot x}$

2) (4 points each) Find the exact value of $\tan\dfrac{\pi}{12}$ using the given methods. Rationalize the denominator in part *a*:

a) A Sum or Difference Formula:

b) A Half Angle Formula:

c) Using your answer from either part *a* or *b* above, explain how you can find the exact value of $\tan\left(\dfrac{13\pi}{12}\right)$ by using $\dfrac{\pi}{12}$ as a reference angle:

3) (5 points) Simplify:

$$\frac{\sin\left(\alpha+\beta\right)-\sin\left(\alpha-\beta\right)}{\cos\left(\alpha+\beta\right)-\cos\left(\alpha-\beta\right)}$$

4) (5 points each) Given $\cos\theta = \frac{7}{25}$, where θ is in Quadrant IV, find the exact values for…

a) $\sin(2\theta)$

b) $\cos(2\theta)$

c) $\tan(2\theta)$

d) The Quadrant where 2θ resides:

5) (5 points) Simplify:

$$\frac{\sin^2\alpha}{\tan^2\alpha} - \frac{\tan^2\alpha}{\sec^2\alpha}$$

6) (4 points each) Simplify or explain why it does not exist:

a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

b) $\cos\left(\cos^{-1}(-1.1)\right)$

c) $\cos^{-1}\left(\cos\left(-\frac{2\pi}{3}\right)\right)$

d) $\sin\left(\sin^{-1}\left(-\frac{4}{5}\right) - \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

7) (6 points each) Solve for the variable:

a) $\cos^2 x - 1 = 0$

b) $\sin(2x) = \frac{\sqrt{3}}{2}$ on $[0, 2\pi)$

8) Fill in the blank using interval notation:

	$\sin x$ *	$\cos x$ *	$\tan x$ *	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain						
Range						

*Write the domain restrictions for these three functions.

9) (2 points) Explain why we restricted the domains of $y = \sin x$, $y = \cos x$, and $y = \tan x$ in this chapter.

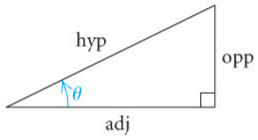
Extra Credit:

Simplify: $-\sin\left(\frac{\pi}{3} - \alpha\right)\sin\left(\frac{\pi}{3} + \alpha\right) + \cos\left(\frac{\pi}{3} - \alpha\right)\cos\left(\frac{\pi}{3} + \alpha\right)$

Trigonometry

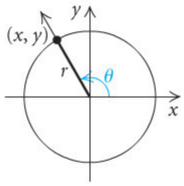
Trigonometric Functions

Acute Angles



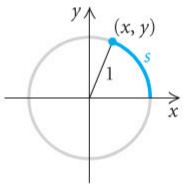
$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \csc\theta = \frac{\text{hyp}}{\text{opp}},$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \sec\theta = \frac{\text{hyp}}{\text{adj}},$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}}, \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Any Angle



$$\sin\theta = \frac{y}{r}, \quad \csc\theta = \frac{r}{y},$$
$$\cos\theta = \frac{x}{r}, \quad \sec\theta = \frac{r}{x},$$
$$\tan\theta = \frac{y}{x}, \quad \cot\theta = \frac{x}{y}$$

Real Numbers



$$\sin s = y, \quad \csc s = \frac{1}{y},$$
$$\cos s = x, \quad \sec s = \frac{1}{x},$$
$$\tan s = \frac{y}{x}, \quad \cot s = \frac{x}{y}$$

Basic Trigonometric Identities

$$\sin(-x) = -\sin x,$$
$$\cos(-x) = \cos x,$$
$$\tan(-x) = -\tan x,$$

$$\tan x = \frac{\sin x}{\cos x},$$
$$\cot x = \frac{\cos x}{\sin x},$$

$$\csc x = \frac{1}{\sin x},$$
$$\sec x = \frac{1}{\cos x},$$
$$\cot x = \frac{1}{\tan x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1,$$
$$1 + \cot^2 x = \csc^2 x,$$
$$1 + \tan^2 x = \sec^2 x$$

Identities Involving $\pi/2$

$$\sin(\pi/2 - x) = \cos x,$$
$$\cos(\pi/2 - x) = \sin x, \quad \sin(x \pm \pi/2) = \pm\cos x,$$
$$\tan(\pi/2 - x) = \cot x, \quad \cos(x \pm \pi/2) = \mp\sin x$$

Sum and Difference Identities

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v,$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v,$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Identities

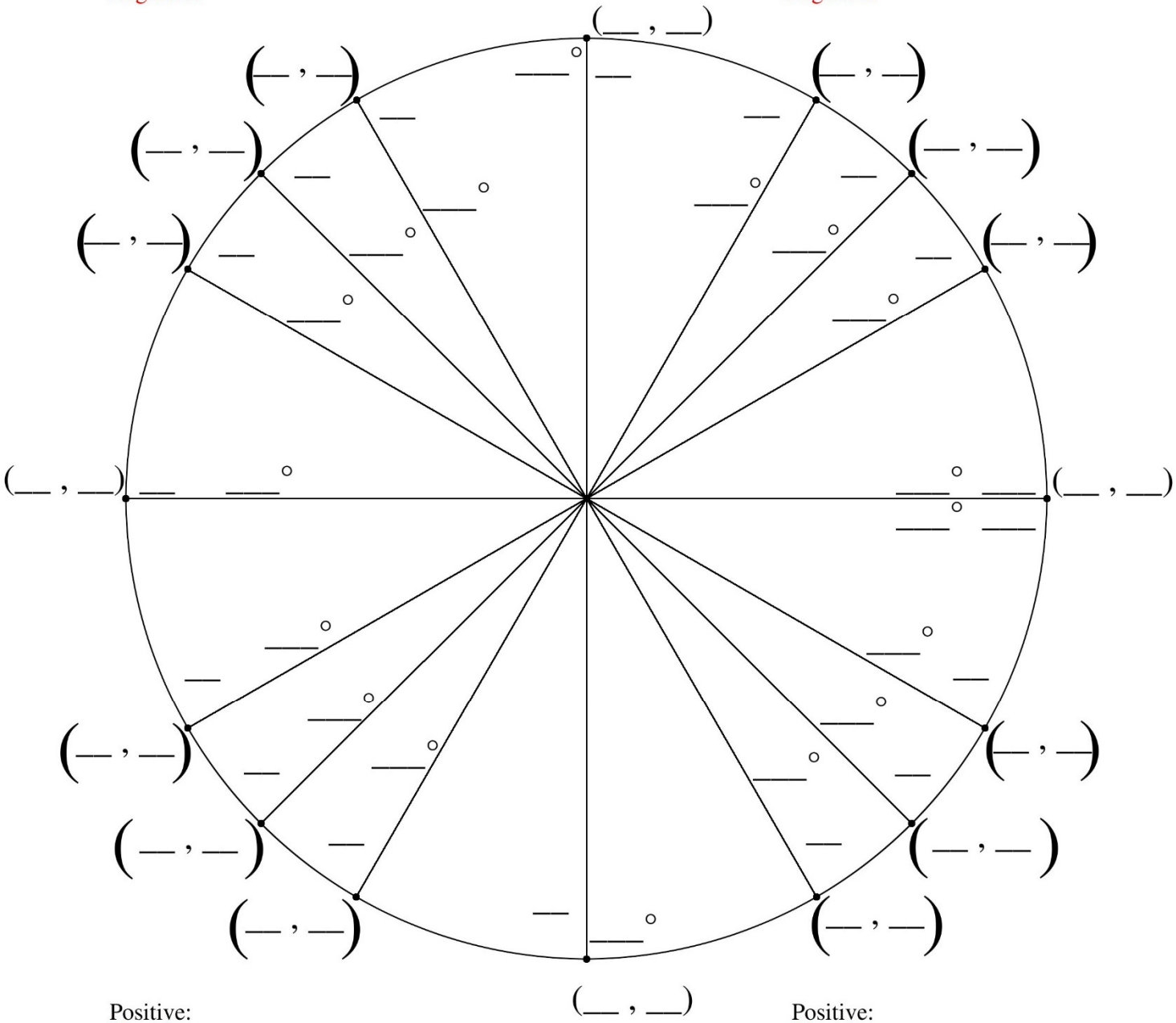
$$\sin 2x = 2 \sin x \cos x,$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1,$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}, \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}},$$
$$\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Positive:
Negative:

Positive:
Negative:



Positive:
Negative:

Positive:
Negative: