1) (3 points) What is a sequence?

a basket of puppies

2) (5 points each) Find the first five terms of the following sequences. Determine if they are arithmetic, geometric, or neither. If it is arithmetic, determine the common difference. If it is geometric, determine the common ratio.

a) $\left\{2(-3)^n\right\}$

$n=1 \quad 2(-3)^1 = -6$

$n=2 \quad 2(-3)^2 = 18$

$n=3 \quad 2(-3)^3 = -54$

$n=4 \quad 2(-3)^4 = 162$

$n=5 \quad 2(-3)^5 = -486$

geometric $r=-3$

b) $a_1 = 3,\ a_{n+1} = a_n + \frac{1}{2},\ n \geq 1$

$a_1 = 3$

$a_2 = a_1 + \frac{1}{2} = 3 + \frac{1}{2} = \frac{7}{2}$

$a_3 = a_2 + \frac{1}{2} = \frac{7}{2} + \frac{1}{2} = 4$

$a_4 = a_3 + \frac{1}{2} = 4 + \frac{1}{2} = \frac{9}{2}$

$a_5 = a_4 + \frac{1}{2} = \frac{9}{2} + \frac{1}{2} = 5$

arithmetic $d = \frac{1}{2}$

3) (4 points) Find the sum $\sum_{k=1}^{6} \frac{k}{3}$. Write out the terms and write answer as an improper fraction:

$= \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \frac{4}{3} + \frac{5}{3} + \frac{6}{3} = \frac{21}{3} = \boxed{7}$

4) (4 points each) Write in sigma notation:

a) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots + \frac{49}{50}$

$$\sum_{k=1}^{49} \frac{k}{k+1}$$

b) $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \ldots + \frac{1}{4,096}$

$$\sum_{k=0}^{6} \left(-\frac{1}{4}\right)^k$$

5) (3 points each) For the arithmetic sequence $-4, -1, 2, 5, 8\ldots$, find and simplify…

a) a_n using $a_n = a_1 + (n-1)d$:

$a_n = -4 + (n-1)(-3)$

$a_n = -3n - 1$

b) a_{412}:

$a_{412} = -3(412) - 1$

$= -1237$

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6) (4 points) Given the arithmetic sequence $\{a_n\}$ where $a_7 = 29$ and $a_{14} = 1$ find a_n using a system of equations and the formula $a_n = a_1 + (n-1)d$. Solve the system using any learned technique:

$a_7 = 29 = a_1 + (7-1)d$

$a_{14} = 1 = a_1 + (14-1)d$

$\Rightarrow \begin{cases} a_1 + 6d = 29 \\ a_1 + 13d = 1 \end{cases}$

$-7d = 28$

$d = -4$

$a_1 + 6(-4) = 29$

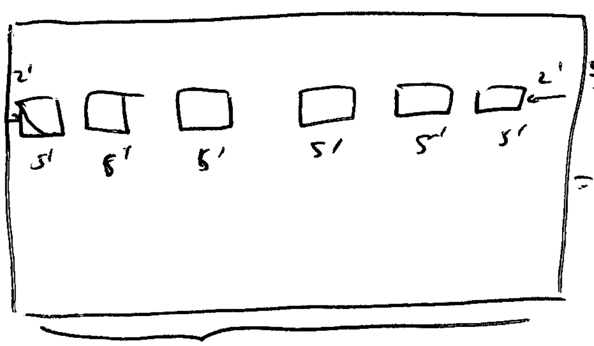
$a_1 = 53$

$a_n = 53 + (n-1)(-4)$

$\boxed{a_n = -4n + 57}$

7) (4 points each) Mike decides to hang his completely awesome Harry Potter posters on a 50 foot wall in his office. The 6 frames are 5 feet long each. He wants the first and last frames to be in from the corner by 2 feet and equal spacing in between each frame.

a) Draw a complete picture for this scenario:



$\frac{50-(5\cdot 6+2\cdot 2)}{5}$

$= 3.2 \text{ ft}$

b) Starting from the left corner, where should the frames go? Count to the left edge of each frame.

2', 10.2', 18.4', 26.6', 34.8', 43'

Common difference $3.2 + 5 = 8.2$

Professor Snape (top right) is shown here using the approved McCraith technique of getting students to pay attention.

c) Assuming the nails will go in the exact horizontal center of the frame, where should Mike put the nails into the wall? Start your count from the left corner:

4.5', 12.7', 20.9', 29.1', 37.3', 45.5'

8) (5 points each) Evaluate the sums using either formula: $S_n = \frac{n}{2}(2a_1 + (n-1)d)$ or $S_n = \frac{n}{2}(a_1 + a_n)$.

Write answers as improper fractions as needed:

a) $\sum_{k=1}^{70}(4x-10)$

$n = 70 \quad a_1 = 4(1) - 10 = -6$

$a_{70} = 4(70) - 10 = 270$

$S_{70} = \frac{70}{2}(-6 + 270)$

$= \boxed{9240}$

b) $\sum_{k=12}^{48}\left(\frac{5k+10}{4}\right)$

$n = 48 - 12 + 1 = 37$

$a_1 = \frac{5(12)+10}{4} = 17.5 \quad a_{37} = \frac{5(48)+10}{4} = 62.5$

$S_{37} = \frac{37}{2}(17.5 + 62.5) = \boxed{1480}$

9) (3 points) Short answer: Why can't you find an infinite sum of terms of an arithmetic sequence, but you can under certain constrictions of a geometrics sequence.

because ice cream?

10) (3 points each) For the geometric sequence $\frac{2}{9}, \frac{2}{3}, 2, 6, \ldots$, find the following given $a_n = a_1 r^{n-1}$:

a) a_n

$\boxed{a_n = \frac{2}{9}(3)^{n-1}}$

b) a_{15}

$a_{15} = \frac{2}{9}(3)^{15-1} = \boxed{1062882}$

11) (4 points each) Using the formulas $S_n = a_1 \frac{1-r^n}{1-r}$ and $S_\infty = \frac{a_1}{1-r}$ (respectively), find by writing answers as an improper fraction…

a) $8 + 2 + \frac{1}{2} + \frac{1}{8} + \ldots + \frac{1}{128}$: $8(\frac{1}{4})^0$ $\to 8(\frac{1}{4})^5$

geometric $a_1 = 8$ $r = \frac{1}{4}$

$n = 6$

$S_6 = 8 \cdot \frac{1-(\frac{1}{4})^6}{1-\frac{1}{4}} = \boxed{\frac{1365}{128}}$

b) $8 + 2 + \frac{1}{2} + \frac{1}{8} + \ldots$

$S_\infty = \frac{8}{1-\frac{1}{4}} = \boxed{\frac{32}{3}}$

12) (4 points) Find the fractional representation of the repeating decimal $9.\overline{5}$. Use the formula $S_\infty = \frac{a_1}{1-r}$ in your work:

$9.\overline{5} = 9.555\ldots = 9 + \underbrace{0.5 + 0.05 + 0.005 + \ldots}$

$= 9 + \frac{0.5}{1-0.1}$

$= 9 + \frac{5}{9} = \boxed{\frac{86}{9}}$

J1

13) (5 points) A ball is dropped from a height of 80 feet and always rebounds ½ of the distance fallen. How far does the ball travel vertically before coming to a stop? In your answer be sure to use the formula $S_\infty = \dfrac{a_1}{1-r}$.

$80 + 40 + 40 + 20 + 20 + \ldots$

$80 + 2\left(\dfrac{40}{1-1/2}\right) = \boxed{240 \text{ ft}}$

14) (5 points) Evaluate the sum $\sum_{k=1}^{\infty} 4\left(\dfrac{1}{3}\right)^{k-1}$ using $S_\infty = \dfrac{a_1}{1-r}$:

$= 4\left(\frac{1}{3}\right)^{1-1} + 4\left(\frac{1}{3}\right)^{2-1} + 4\left(\frac{1}{3}\right)^{3-1} + \ldots$

$= 4 + \frac{4}{3} + \frac{4}{9} + \ldots$

$a_1 = 4 \quad r = \frac{1}{3}$

$S_\infty = \dfrac{4}{1-\frac{1}{3}} = \boxed{6}$