1) (10 points) Solve the system $\begin{cases} 3x-y=2 \\ x-2y+2z=-2 \\ 2x-3y+3z=-1 \end{cases}$ using matrix inverses. Be sure to show the work necessary to obtain the inverse.

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 0 & 1 & 0 & 0 \\ 1 & -2 & 2 & 0 & 1 & 0 \\ 2 & -3 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-2R_1+R_3\to R_3]{R_1\leftrightarrow R_2,\ -3R_1+R_2\to R_2} \left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -6 & 1 & -3 & 0 \\ 0 & 1 & -1 & 0 & -2 & 1 \end{array}\right] \xrightarrow[-5R_2+R_3\to R_3]{R_2\leftrightarrow R_3,\ 2R_2+R_1\to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & -1 & 0 & -2 & 1 \\ 0 & 0 & -1 & 1 & 7 & -5 \end{array}\right]$$

$$\xrightarrow[R_3+R_2\to R_2]{-R_3\to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & -1 & -9 & 6 \\ 0 & 0 & 1 & -1 & -7 & 5 \end{array}\right]$$

$$\begin{bmatrix} 0 & -3 & 2 \\ -1 & -9 & 6 \\ -1 & -7 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \\ 7 \end{bmatrix} \begin{matrix} x \\ y \\ z \end{matrix}$$

2) (3 points) Prove that the matrix $\begin{bmatrix} 4 & 0 \\ 3 & 0 \end{bmatrix}$ has no inverse. Hint: Assume $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ is the inverse and show that this assumption leads to a contradiction.

$$\begin{bmatrix} 4 & 0 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4a & 4b \\ 3a & 3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

If we were to multiply, we should get $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Well, we can't have $4a=1$ and also $3a=0$ at the same time... NO inverse!

3) (10 points each) Decompose into partial fractions:

a) $\dfrac{x+10}{x^2-4x-12} = \dfrac{A}{x-6} + \dfrac{B}{x+2}$

$x+10 = A(x+2) + B(x-6)$

$x+10 = Ax+2A+Bx-6B$

$\begin{cases} A+B=1 \\ 2A-6B=10 \end{cases} \Rightarrow \begin{matrix} A=2 \\ B=-1 \end{matrix}$

$$\boxed{\frac{2}{x-6} + \frac{-1}{x+2}}$$

b) $\dfrac{x^2+2x+7}{(x^2+2)(x+1)} = \dfrac{Ax+B}{x^2+2} + \dfrac{C}{x+1}$

$x^2+2x+7 = (Ax+B)(x+1) + C(x^2+2)$

$= Ax^2+Bx+Ax+B+Cx^2+2C$

$\begin{cases} A+C=1 \\ A+B=2 \\ B+2C=7 \end{cases} \Rightarrow \begin{matrix} A=-1 \\ B=3 \\ C=2 \end{matrix}$

$$\frac{-x+3}{x^2+2} + \frac{2}{x+1}$$

33

4) (8 points each) Fill in the information for the following parabolas:

a) $y^2 = 36x$: $4p = 36 \Rightarrow p = 9$

i) $h =$ 0

ii) $k =$ 0

iii) $p =$ 9

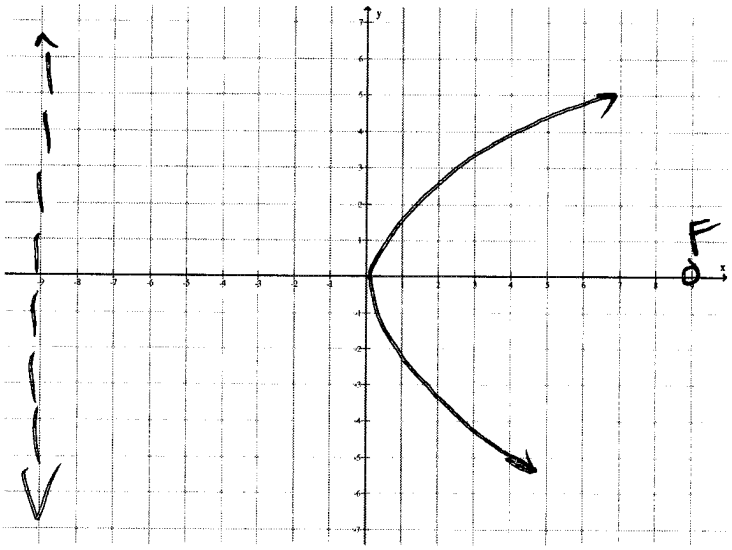
Write iv and v as ordered pairs:

iv) Center (0, 0)

v) Focus (9, 0)

vi) Directrix $x = -9$

vii) Sketch the graph:



b) $(y-2)^2 = -12(x+1)$: $4p = -12$, $p = -3$

i) $h =$ −1

ii) $k =$ 2

iii) $p =$ −3

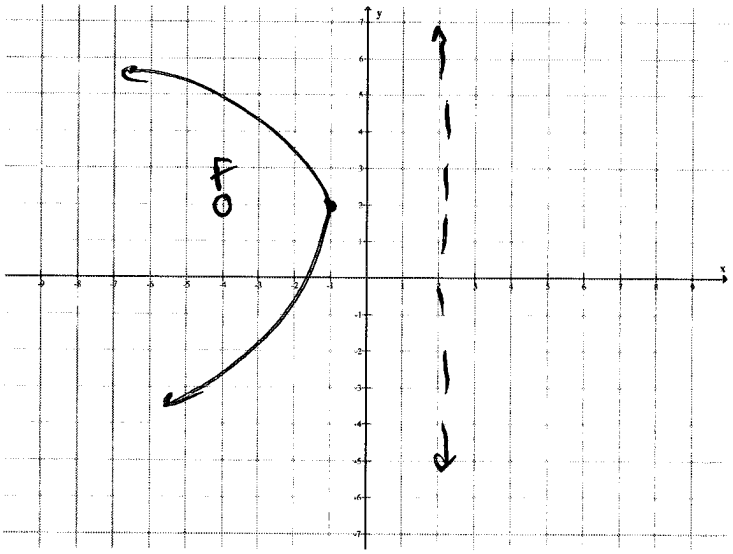
Write iv and v as ordered pairs:

iv) Center (−1, 2)

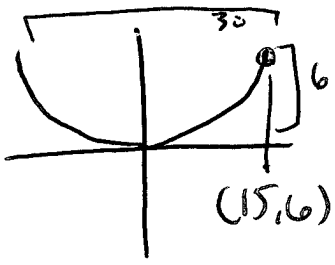
v) Focus (−4, 2)

vi) Directrix $x = 2$

vi) Sketch the graph:



5) (5 points) Determine the location of the receiver for a 30 foot tall, 6 foot deep parabolic satellite dish assuming the receiver is to be at installed at the foci of the parabola. Give your answer in feet from the vertex of the dish:



$x^2 = 4py$

$15^2 = 4p(6) \Rightarrow p = \frac{225}{24} = 9.375'$

Can be drawn vertically or horizontally

21

6) (6 points each) For the ellipse $25x^2 - 100x + 16y^2 + 96y + 244 = 400$

a) Rewrite the equation by completing the square for both variables:

$25(x^2 - 4x + 4) + 16(y^2 + 6y + 9) = 156 + 100 + 144$

$25(x-2)^2 + 16(y+3)^2 = 400$

$$\frac{(x-2)^2}{16} + \frac{(y+3)^2}{25} = 1$$

b) Find the **exact values** for the following:

i) $h =$ 2

ii) $k =$ -3

iii) $a =$ 5

iv) $b =$ 4

v) $c =$ 3

Write as ordered pairs:

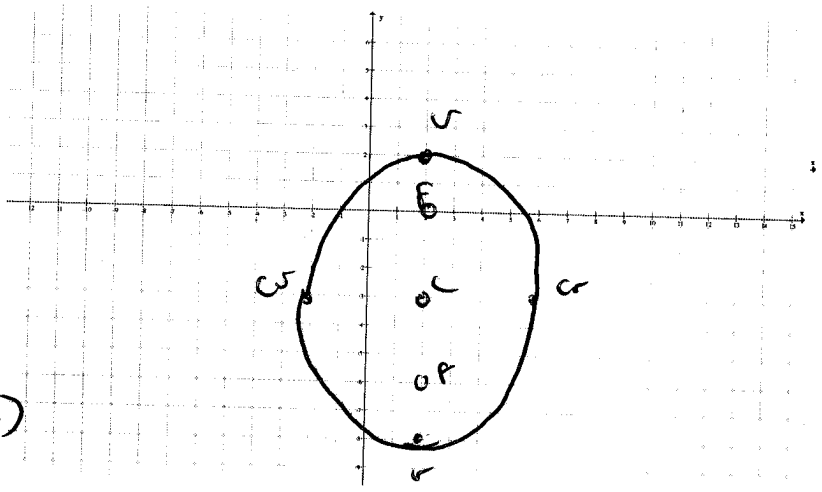
vi) Center (2, -3)

vii) Vertices (2, 2) (2, -8)

viii) Co-Vertices (6, -3) (-2, -3)

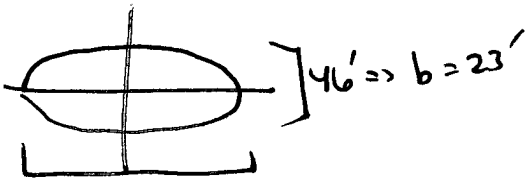
ix) Foci (2, 0) (2, -6)

c) Sketch the graph:



7) (4 points each) Statuary Hall, also known as the Whispering Gallery, is an elliptical room in the United States Capitol in Washington D.C. where a person standing at one focus of the room can hear even a whisper spoken by a person standing at the other focus. Statuary Hall is 46 feet wide and 97 feet long. Assuming a horizontal ellipse, find…

a) The equation of the ellipse for the room:



46' => b = 23'

97' => a = 48.5'

$$\frac{x^2}{48.5^2} + \frac{y^2}{23^2} = 1$$

or 2352.25 or 529

b) The location of the foci from the center of the room. Round to two decimal places:

$c^2 = a^2 - b^2$

$c^2 = 2352.25 - 529$

$\Rightarrow c \approx 42.70$ ft

20

8) (8 points) Fill in the information for the hyperbola $\frac{(x-2)^2}{9}-\frac{(y+1)^2}{16}=1$:

i) $h=$ 2

ii) $k=$ -1

iii) $a=$ 3

iv) $b=$ 4

v) $c=$ 5

Write as ordered pairs:

vi) Center (2, -1)

vii) Vertices (5, -1) (-1, -1)

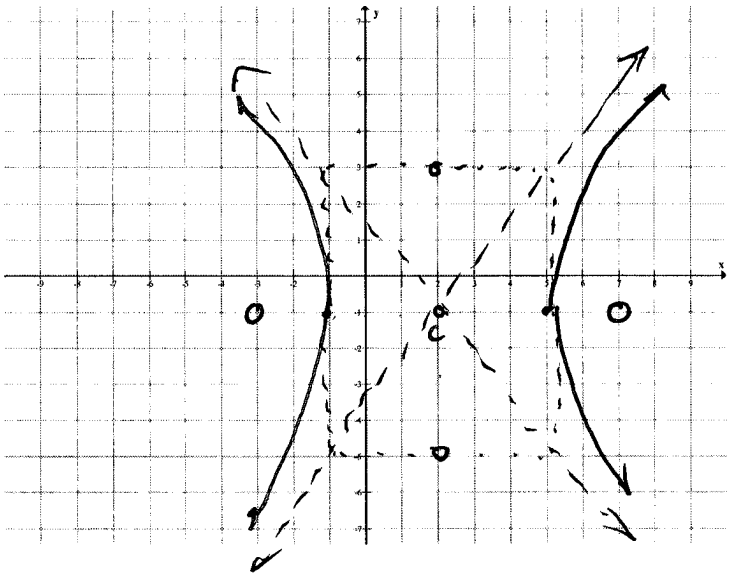
viii) Co-Vertices (2, 3) (2, -5)

ix) Foci (7, -1) (-3, -1)

x) Asymptotes $y=\frac{4}{3}x-\frac{11}{3}, -\frac{4}{3}x+\frac{5}{3}$

$y-(-1)=\pm\frac{4}{3}(x-2) \Rightarrow$

vi) Sketch the graph:



9) (6 points each) Solve the following systems. For part *b*, shade your final in the darkest:

a) $\begin{cases}(x-3)^2+(y+1)^2=16\\ x+3y=4 \Rightarrow x=4-3y\end{cases}$

$(4-3y-3)^2+(y+1)^2=16$

$(1-3y)^2+(y+1)^2=16$

$9y^2-6y+1+y^2+2y+1=16$

$10y^2-4y-14=0$

$5y^2-2y-7=0$ $\quad (y+1)(5y-7)=0$

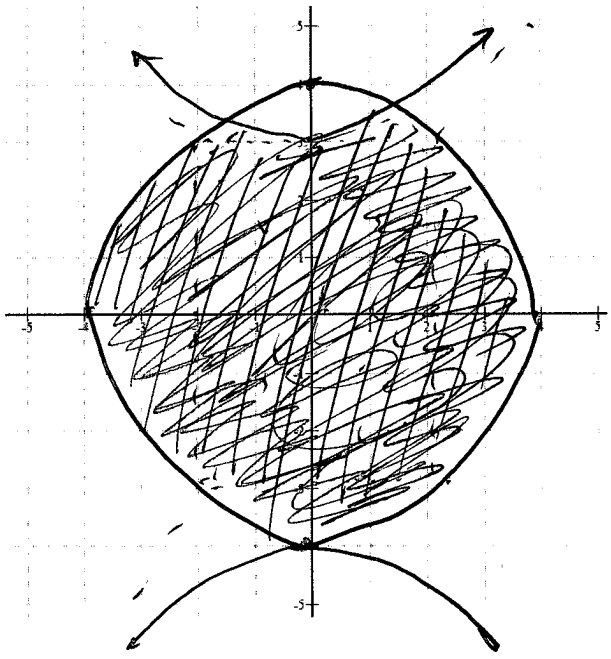
$y=-1$ $\quad y=7/5$

$x=4-3y$ $\quad x=4-3(7/5)$

$x=4-3(-1)=7$ $\quad =-1/5$

$(7,-1)$ $\quad \left(-\frac{1}{5},\frac{7}{5}\right)$

b) $\begin{cases}x^2+y^2\le 16 & \text{circle}\\ \frac{y^2}{9}-\frac{x^2}{4}\le 1 & \text{hyperbola}\end{cases}$



29