1) (3 points each) Find the inverse of the following functions:

a) $\{(6,-2),(-5,8),(0,-9),(16,1)\}$

$\{(-2,6),(8,-5),(-9,0),(1,16)\}$

b) $f(x)=\dfrac{x+7}{x-4}$

① $y=\dfrac{x+7}{x-4}$

② $x=\dfrac{y+7}{y-4}$

③ $xy-4x=y+7$

$xy-y=4x+7$

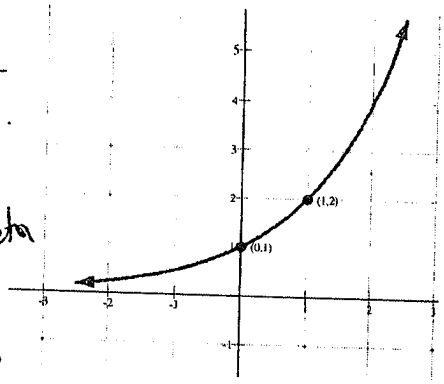
$y(x-1)=4x+7$

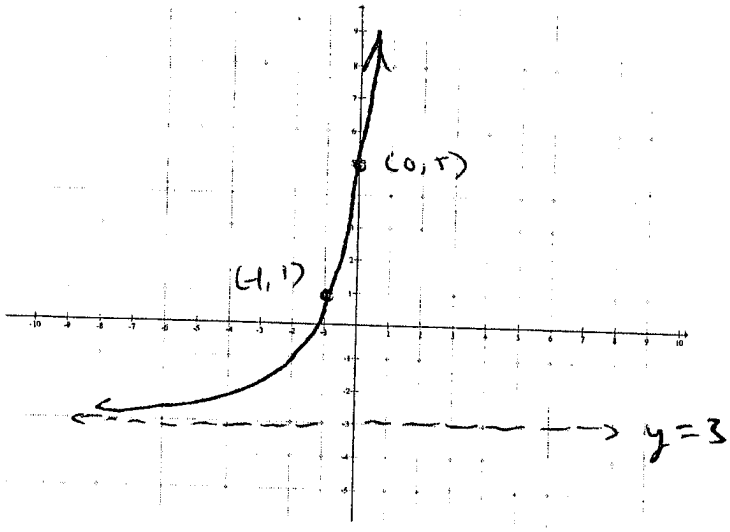
$y=\dfrac{4x+7}{x-1}$

④ $f^{-1}(x)=\dfrac{4x+7}{x-1}$

2) (3 points) Sketch the graph of the function $f(x)=4\cdot 2^{x+1}-3$. Be sure to label the transformed given points and asymptote:

1) left 1
2) vert. stretch by a factor of 4
3) down 3





3) (4 points) Write the expression as one logarithm: $5\log x+9\log y-\dfrac{1}{3}\log z$

$\log x^5+\log y^9-\log z^{1/3}$

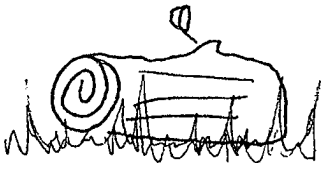
$\log\dfrac{x^5y^9}{z^{1/3}}$

4) (4 points) Given that $\log_a x=3$, $\log_a y=-4$, and $\log_a z=5$, find the value of $\log_a\sqrt{\dfrac{x^3y^5}{z^2}}$:

$\log_a\sqrt{\frac{x^3y^5}{z^2}}=\frac{1}{2}\log_a\left(\frac{x^3y^5}{z^2}\right)=\frac{1}{2}\left[\log_a x^3+\log_a y^5-\log_a z^2\right]$

$=\frac{1}{2}\left[3\log_a x+5\log_a y-2\log_a z\right]=\frac{1}{2}\left[3\cdot 3+5\cdot(-4)-2(5)\right]=\boxed{-\frac{21}{2}}$

5) (2 points) Short answer: Why are logarithms necessary?



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6) (3 points each) Simplify completely:

a) $\log_{25}(5x)$

$= \log_{25} 5 + \log_{25} x$

$= \boxed{1/2 + \log_{25} x}$

b) $\ln\left(\frac{e^9}{x^2}\right)$

$= \ln e^9 - \ln x^2$

$= \boxed{9 - 2\ln x}$

c) $\log_2 25 \times \log_5 8$

$\frac{\log 25}{\log 2} \cdot \frac{\log 8}{\log 5} = \frac{\log 25}{\log 5} \cdot \frac{\log 8}{\log 2}$

$= \log_5 25 \cdot \log_2 8 = 2 \cdot 3 = \boxed{6}$

7) (5 points each) Solve for the variable.

a) $4^{3x+7} = 64$

$4^{3x+7} = 4^3$

$3x+7 = 3 \Rightarrow \boxed{x = \frac{-4}{3}}$

b) $5^{3x-2} = 35$

$\log_5 35 = 3x - 2 \Rightarrow$

$\boxed{x = \frac{\log_5 35 + 2}{3}}$

c) $\ln(4x-7) - \ln(x-2) = \ln(4)$

$\ln\left(\frac{4x-7}{x-2}\right) = \ln 4$

$\frac{4x-7}{x-2} = 4 \Rightarrow 4x - 7 = 4x - 8$

$-7 = -8$ $\boxed{\emptyset}$

d) $\log_2(5x+15) - \log_2(x+2) = 3$

$\log_2 \frac{5x+15}{x+2} = 3$

$\frac{5x+15}{x+2} = 2^3 = 8$

$5x + 15 = 8x + 16 \Rightarrow \boxed{x = -\frac{1}{3}}$

e) $2e^x - 9 = -4e^{-x}$

$2e^x - 9 + \frac{4}{e^x} = 0$

$2(e^x)^2 - 9e^x + 4 = 0$

$(2e^x - 1)(e^x - 4) = 0$

$e^x = \frac{1}{2} \Rightarrow \boxed{x = \ln\frac{1}{2}}$ $\quad e^x = 4 \Rightarrow \boxed{x = \ln 4}$

8) (5 points) At a press conference, Microsoft announced they were dropping the price of the Xbox One to make it more competitive this holiday season. PlayStation fans quickly responded with "TROLOL" and the number of laughing fans has been increasing exponentially, beginning with 12.4 million fans. Using $P(t) = P_0 e^{kt}$ where P is in **millions** and t is the number of days since the announcement, determine the exact value for the growth rate k if after 6 days, there were 23.8 million people fans laughing.

$23.8 = 12.4\, e^{k(6)} \Rightarrow \frac{238}{124} = e^{6k}$

$\Rightarrow \ln\frac{238}{124} = 6k \Rightarrow \boxed{k = \frac{\ln\left(\frac{238}{124}\right)}{6}}$ reduces to $\frac{\ln\frac{119}{62}}{6}$

9) (10 points each) Solve the system $\begin{cases} x+2y+3z=1 \\ -2x-3y-6z=-3 \\ 3x+6y+10z=6 \end{cases}$ using the methods listed below. Be sure to show all necessary work where appropriate. Write answer as an ordered triple:

a) Elimination Method:

$\times(2)$ $\quad 2x+4y+6z=2$

$-2x-3y-6z=-3$

$y = -1$

$\times(-3)$ $\quad -3x-6y-9z=-3$

$3x+6y+10z=6$

$z=3$

$x+2y+3z=1$

$x+2(-1)+3(3)=1$

$x=-6$

$(-6,-1,3)$

b) Gauss-Jordan Method:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ -2 & -3 & -6 & -3 \\ 3 & 6 & 10 & 6 \end{array}\right] \xrightarrow{\substack{2R_1+R_2\to R_2 \\ -3R_1+R_3\to R_3}} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{\substack{-2R_2+R_1\to R_1 \\ -3R_3+R_1\to R_1}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

10) (2 points) Verify that you made absolutely sure that your answer to 9a is the same as in 9b by signing your name here Bellamy (Henry P.). You will not receive the credit if the work does not support the same answer.

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WHERE'S THE GIRL?

11) (6 points) **Set up but do not solve** the system associated with the following. **Be sure to name and define your variables:**

"Geez, Batman. I'm right here in the game *Detroit*."

At a recent conference, Sony unveiled three new games: *Detroit*, *Dreams*, and *Horizon Zero Dawn*. Unable to resist preordering multiple copies of each game for himself and his friends, Mike goes wild during an online-shopping binge. At Target, Mike orders 3 copies of *Detroit*, 4 copies of *Dreams*, and 2 copies of *Horizon Zero Dawn* and spends \$150.44. At Best Buy, Mike orders 4 copies of *Detroit*, 5 copies of *Dreams*, and 1 copy of *Horizon Zero Dawn* and spends \$195.44. The price for one copy of *Detroit* is half of the other two games combined. Assume the prices for each game does not change depending on where the game was purchase. How much does each game cost?

x = price of Detroit

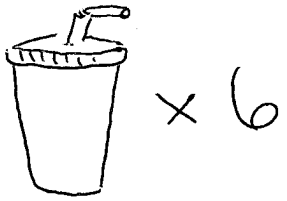
y = price of Dream

z = price of Horizon Zero Dawn

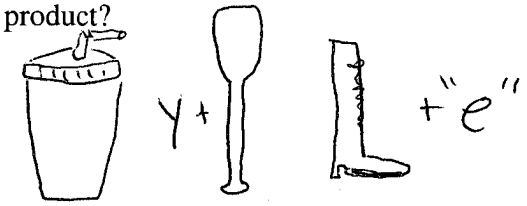
$$\begin{cases} 3x+4y+2z=150.44 \\ 4x+5y+z=195.44 \\ x=\frac{1}{2}(y+z) \text{ or } x-\frac{1}{2}y-\frac{1}{2}z=0 \\ \text{or } 2x=y+z \end{cases}$$

12) (2 points) Short answer. Do not provide an example as your answer:

a) What properties must be true about the matrices A and B in order to find their sum or difference?



b) What property must be true about the matrices A and B in order to find their product?



13) (3 points each) For the given matrices, find the following or explain why they do not exist:

$$A=\begin{bmatrix} 4 & 2 & 0 \\ 1 & -3 & 5 \end{bmatrix} \qquad B=\begin{bmatrix} 6 & 2 & -1 \\ 9 & 0 & 3 \end{bmatrix} \qquad C=\begin{bmatrix} 4 & -2 \\ 0 & 1 \end{bmatrix}$$

a) $3A-5B$

$$\begin{bmatrix} -18 & -4 & 5 \\ -42 & -9 & 0 \end{bmatrix}$$

b) $7B+C$

Nope! Not the same dimension.

c) CB

$$\begin{bmatrix} 6 & 8 & -10 \\ 9 & 0 & 3 \end{bmatrix}$$

14) (1 measly point) Fill in the blank: John Jacob Jingleheimer Schmidt, a foreign exchange student from Finland, is in your math class. (Yes, in the future, you'll be teaching math—kudos: me.) He asks one day for you to pronounce $\log_6 5$ for him. You reply "Gladly, it's pronounced _____"

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