1) (2 points) Solve for the variable: $x^4+34x^2+225=0$:

$(x^2+9)(x^2+25)=0$

$x^2=-9$ $\quad$ $x^2=-25$

$\boxed{x=\pm 3i \qquad x=\pm 5i}$

2) (1 point each) For the function $f(x)=2x^2+6x+9$, determine…

a) If it opens up or down. How do you know?

Up $a=2>0$

b) The coordinates of the vertex:

$x=-\frac{b}{2a}=-\frac{6}{2(2)}=-\frac{3}{2}$

$f(-\frac{3}{2})=\frac{9}{2}$

$\boxed{\left(-\frac{3}{2},\frac{9}{2}\right)}$

c) The domain: $\mathbb{R}$

d) The range: $\left[\frac{9}{2},\infty\right)$

e) Interval of increase: $\left(-\frac{3}{2},\infty\right)$

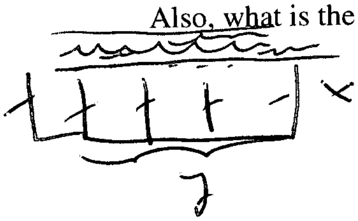
f) Interval of decrease: $\left(-\infty,-\frac{3}{2}\right)$

g) Based only on your answers to parts *a* and *b*, will it have *x*-intercepts? Why or why not?

No: 1) It opens up
2) vertex is above the x-axis.

#sadpanda

3) (4 points) A farmer has 500 feet of fence and wishes to enclose 4 adjacent rectangular pens that are all next to a river. The side against the river will not receive any fencing. Determine the dimensions of the enclosure to maximize its area. Let *x* represent the width of that enclosure. Also, what is the maximum area?



$5x+y=500 \Rightarrow y=500-5x$

$A(x)=x(500-5x)=-5x^2+500x$

$x=-\frac{b}{2a}=-\frac{500}{2(-5)}=50'$ $\quad$ $y=500-5(50)=250'$

$\boxed{A(50)=12{,}500 \text{ sq ft}}$

4) (2 points each) Solve for the variable. Write part *b* in interval notation:

a) $(x-3)\,x+\frac{12(x-3)}{x-3}=1(x-3)+\frac{4x}{x-3}(x-3)$

$x(x-3)+12=x-3+4x$

$x^2-3x+12=5x-3$

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$\cancel{x=3}$ $\quad$ $x=5$

b) $6|2x+1|-7\le 10$

$|2x+1|\le\frac{17}{6}$

$-\frac{17}{6}\le 2x+1\le\frac{17}{6}$

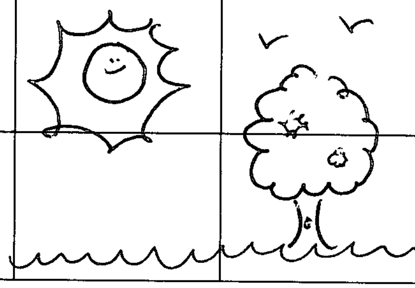
$-\frac{23}{12}\le x\le\frac{11}{12}$

$\boxed{\left[-\frac{23}{12},\frac{11}{12}\right]}$

17

5) (2 points) Fill in the chart with a sketch of the location of the arrowheads:

	Even Degree	Odd Degree
Positive Leading Coefficient		
Negative Leading Coefficient		



6) (3 points each) For the function $f(x)=(x+3)^2(2x+1)^2(x-2)\ldots$

a) What is the leading term and which quadrants will the arrowheads end up in? Explain why.

LT: $4x^5$

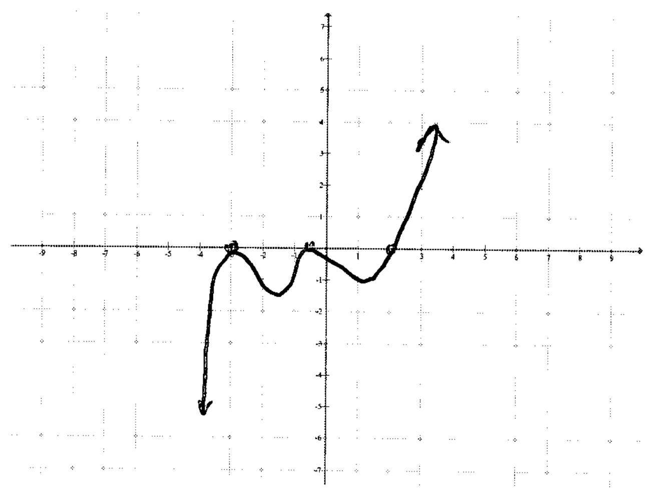
Quad I & III

5 is odd, $4>0$

b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
-3	2	T
$-1/2$	2	T
2	1	C

c) Sketch the graph based on parts *a* and *b*:



7) (2 points each) Form a polynomial function of degree four that meets the following requirements. **Be sure to leave your answer in factored form**:

a) Has zeros at 5, -8, 6, and 12:

$y=(x-5)(x+8)(x-6)(x-12)$

b) Has the same zeros and multiplicity as in part *a* but is a different function:

$y=\frac{2\pi}{e^2}(x-5)(x+8)(x-6)(x-12)$

c) Has zeros including $2-7i$ and $12+\sqrt{3}$:

$y=(x-(2-7i))(x-(2+7i))(x-(12+\sqrt{3}))(x-(12-\sqrt{3}))$

17

8) (3 points *a;* 2 points others) Consider the functions $f(x)=2x^3+x^2-7x-26$ and $g(x)=x^2+3x+4$.

a) Divide $f(x)$ by $g(x)$ using long division:

$$
\begin{array}{r}
2x-5 \\
x^2+3x+4 \,\overline{)\,2x^3+x^2-7x-26} \\
-(2x^3+6x^2+8x) \\
\hline
-5x^2-15x-26 \\
-(-5x^2-15x-20) \\
\hline
-6
\end{array}
$$

b) Based on your work in part *a*, was $g(x)$ a factor of $f(x)$? Why or why not?

No, remainder wasn't zero!

c) What is the equation of the oblique asymptote of the rational function $y=\dfrac{2x^3+x^2-7x-26}{x^2+3x+4}$?

$\boxed{y=2x-5}$ ← fancy!

9) (7 points each) Factor the polynomial completely by first listing the possible rational roots and then using synthetic division and your calculator.

a) $f(x)=3x^3+16x^2+15x-18$

$p=\pm 1,2,3,6,9,18 \quad q=\pm 1,3$

$\frac{p}{q}=\pm 1,2,3,6,9,18,\frac{1}{3},\frac{2}{3}$

$$
\begin{array}{r|rrrr}
-3 & 3 & 16 & 15 & -18 \\
 & \downarrow & -9 & -21 & 18 \\
\hline
-3 & 3 & 7 & -6 & \lfloor 0 \\
 & \downarrow & -9 & 6 & \\
\hline
 & 3 & -2 & \lfloor 0 &
\end{array}
$$

$\boxed{f(x)=(x+3)^2(3x-2)}$

b) $g(x)=x^5-11x^4+49x^3-111x^2+128x-60$

$p=\pm 1,2,3,4,5,6,10,12,15,20,30,60$

$q=\pm 1$

$\frac{p}{q}=\pm 1,2,3,4,5,6,10,12,15,20,30,60$

$$
\begin{array}{r|rrrrrr}
2 & 1 & -11 & 49 & -111 & 128 & -60 \\
 & \downarrow & 2 & -18 & 62 & -98 & 60 \\
\hline
2 & 1 & -9 & 31 & -49 & 30 & \lfloor 0 \\
 & \downarrow & 2 & -14 & 34 & -30 & \\
\hline
3 & 1 & -7 & 17 & -15 & \lfloor 0 & \\
 & \downarrow & 3 & -12 & 15 & & \\
\hline
 & 1 & -4 & 5 & \lfloor 0 & &
\end{array}
$$

$g(x)=(x-2)^2(x-3)(x^2-4x+5) \neq \boxed{(x-2)^2(x-3)(x-(2+i))(x-(2-i))}$

21

10) (3 points each) Using your factorized result from 9a, complete the following for the function $f(x) = 3x^3 + 16x^2 + 15x - 18$: $(x+3)^2(3x-2)$

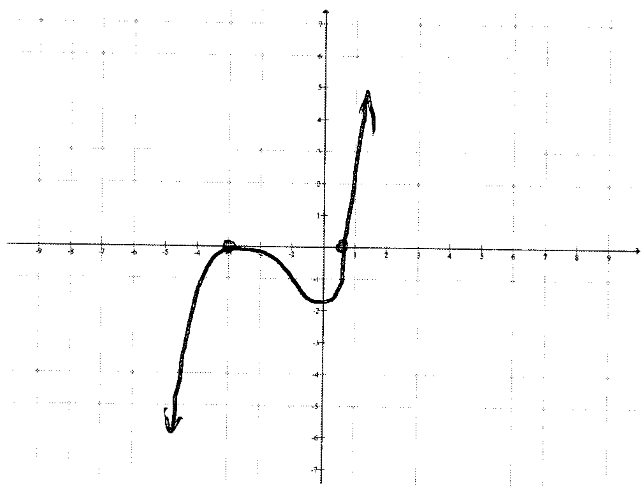
a) What is the leading term and which quadrants will the arrowheads end up in? Explain why.

LT : $3x^3$

Quad I & III

$3 > 0$ 3 is odd

c) Sketch the graph based on parts *a* and *b*:



b) Fill in the chart:

Zero	Multiplicity	Touch/Cross
-3	2	T
$\frac{2}{3}$	1	C

11) (5 points each) For the function $f(x) = \dfrac{2x+2}{x^2-1}$, find…

a) The domain:

$x^2 - 1 = 0$

$x \neq \pm 1$

b) The intercepts (if any):

x-int

$2x+2=0 \Rightarrow x=-1$

$\emptyset$ -1 not in domain

y-int

$f(0) = \frac{2}{-1} = -2$ $(0,-2)$

c) Any vertical asymptotes and holes:

$x=1$

$2(1)+2 \neq 0$

$x=1$ VA

$x=-1$

$2(-1)+2=0$

$\frac{2(\cancel{x+1})}{(\cancel{x+1})(x-1)} = \frac{2}{x-1}$

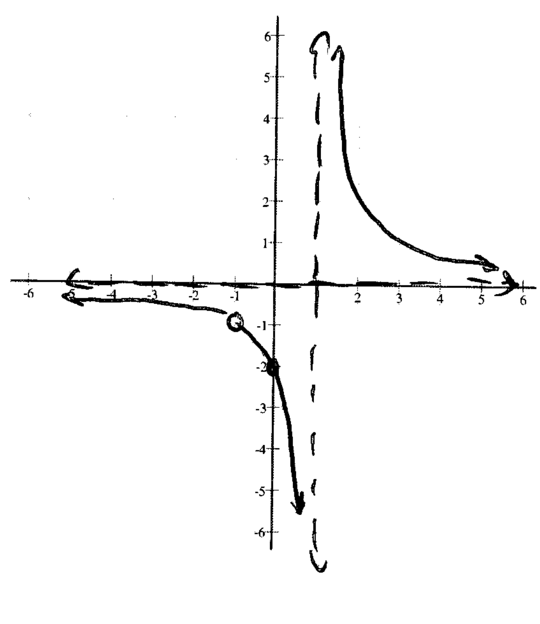
@ $x=-1$ $\frac{2}{-1-1} = -1$

Hole $(-1,-1)$

d) Any horizontal or oblique asymptotes:

HA $y=0$!

e) Sketch a graph using the above information.
HINT: Consider transformations based on the simplified version of the function!



12) (2 points each blank) Fill in the blank:

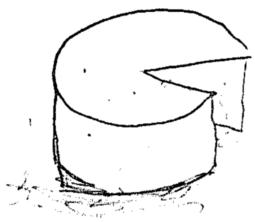
a) If c is a zero of a function f, then $f(c) =$ Timberdoodle, and Barely Buzzed is a factor.

b) Numbers not in the domain of a rational function lead to Cherhive.

13) (3 points each) Short answer. Clearly explain how to find the following algebraically:

a) Vertical Asymptotes and Holes:

b) Horizontal and Oblique Asymptotes:



I Googled funny cheese names.

14) (2 points each) Solve for the variable. Write answer in interval notation:

a) $x^3 - x^2 - 6x \le 0$

$x(x-3)(x+2)=0$

$x = 0, 3, -2$

$-\quad +\quad -\quad +$

$-2 \quad 0 \quad 3$

$(-\infty, -2] \cup [0, 3]$

b) $\dfrac{x-3}{(x-4)(x+2)} \ge 0$

$x-3=0 \Rightarrow x=3$ $\quad (x-4)(x+2)=0 \Rightarrow x=4,-2$

$-\quad +\quad -\quad +$

$-2 \quad 3 \quad 4$

$(-2, 3] \cup (4, \infty)$

Extra Credit (2 points) :

Find the equation of a rational function **in factored form** that has the following properties:

a) Hole at $x = 7$

b) Vertical Asymptotes at $x = -3$ and $x = -8$

c) x-intercepts at $x = \dfrac{8}{3}$ and $x = 2$

d) Horizontal asymptote at $y = 3$

HMM

16/18